Impact of Physicians' Prescribing Behaviour on Hospital Inventory Management Systems

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The study investigates the involvement of physicians in shaping the inventory management systems in a healthcare setting. The physicians' prescribing behaviour for medicines is predicted and decisions regarding when and how much of medicines to order under different conditions and constraints by incorporating uncertainties in medicine prescriptions is modelled using Markovian decision processes. As a case study, a multispecialty hospital in India is considered. The proposed model results in an optimal inventory control policy for medicines with patient and physician demand fulfillment. This complex modelling ties the satisfaction of all the major stakeholders in a healthcare system.

Keywords: Healthcare, Inventory, Physician, Prescribing Behaviour, Medicines, Markov

1. Introduction

Physicians are heterogeneous in nature and show evidence of dynamic prescribing behaviour due to clinical uncertainty, variation in patient characteristics, the difference in physicians' practicing styles, knowledge and experience, pharmaceutical detailing and sampling, etc. (Beam et al. 2017; Montoya, Netzer, and Jedidi 2010). As physicians decide on the appropriate treatment and recommend medicine dosage to be used, they are the essential generators of demand for medicines (Abdulsalam et al. 2018). However, many of the characteristics of physicians' recommendation that determine the appropriate treatment and prescription of medicines are unobserved or partially observed (Montoya, Netzer, and Jedidi 2010; Rappold et al. 2011). In this study, the partially observed states correspond to prescription-behaviour states (for example, high prescription-behaviour state and low prescription-behaviour state). With time physicians may switch among these states through a Markovian process (Montoya, Netzer, and Jedidi 2010). The transitions among the states may be considered dependent on the patient conditions (Beam et al. 2017). It has been observed that in many instances, the inventory of medicines to be used may be dependent on physician dynamics (Abdulsalam et al. 2018). Hence, the impact of the physician dynamics on the demand process of the medicines that subsequently affects the inventory decisions of the hospital needs to be studied so as to access the impact of such dynamics on the inventory level over time.

Physicians interact with the patients, diagnose them and prescribe medicines based on their diagnosis, treatment stages, length-of-stay and type of care unit (Beam et al. 2017; Vila-parrish, Ivy, and King 2008). The physicians also interact with the healthcare facility provider for making decisions related to drug formulary, storage space, availability of medicines, the budget allocated for medicines and service level (Abdulsalam et al. 2018; Little and Coughlan 2008). These interactions may influence physicians' prescription behaviour. In addition, there are various external influencing factors, such as interactions with the pharmaceutical companies, and their marketing activities (for example, detailing and sampling) that leads to variation and uncertainty in physicians' prescription behaviour (Montoya, Netzer, and Jedidi 2010).

The role of the pharmacists or hospital inventory manager is to place the medication order to the pharmaceutical companies or suppliers based on the demand generated from the physicians for the treatment of the patient. However, the pharmacists or hospital inventory manager may only observe information related to the patients and healthcare facilities. They are unable to observe the physicians' prescription behaviour. Therefore, for the pharmacists, the physicians' prescription behaviour is hidden. This partial observation and incomplete information complicate the management of inventory of medicines in the healthcare systems (Arifoğ lu and Özekici 2010a). Literature addresses different types of problems associated with inventory management of medicines in healthcare systems. However, very few studies and research are reported wherein the role of the physician in inventory management is explicitly considered.

According to the World Health Organization, the expenditure on medicines is one of the foremost causes following the growth of healthcare expenditure (World Health Statistics 2018; Lu et al. 2011). To control this growth, healthcare policymakers promote generic medicines. It has been observed that reducing price is not enough to reduce medical expenditures. If physicians prescribe and patients consume more expensive medicines, it may lead to growth in pharmaceutical expenditures. In order to understand the causes behind the growth of pharmaceutical expenditures, it is imperative to investigate the progression of decisions made by physicians, patients, and pharmacists during the treatment process (OECD 2015).

In a medicine inventory management system, there exist numerous causes of uncertainty and variability (Vila-Parrish et al. 2012). Commonly, the existing literature assumes the inventory system to operate in a stationary environment (Uthayakumar and Priyan 2013; Gebicki et al. 2014). Nevertheless, inventory management in healthcare systems is to a certain extent susceptible to variations in the surrounding environment consisting of various patient conditions, heterogeneous physician prescription behaviour, and other factors affecting the demand of medication (Beam et al. 2017; Vila-parrish, Ivy, and King

2008; Vila-Parrish et al. 2012). The existing environmental conditions may vary randomly with their impact on the model parameters. For practical healthcare inventory models, it is apt to consider the likely outcome of various environmental factors on the demand, supply, and inventory-related cost parameters (Yin and Rajaram 2007).

However, heterogeneity among physicians and the dynamics in their prescription behaviour may lead to a continuing impact on the demand of pharmaceuticals (Montoya, Netzer, and Jedidi 2007). Ignoring such behaviour may result in misleading inferences concerning the inventory decisions. Hence, partially observed Markov decision process (POMDP) approach for medicine inventory control is necessary for fulfilling the above-mentioned conditions (Montoya, Netzer, and Jedidi 2010). The model allows considering the impact of randomly changing environment on medicine demand that influences the inventory replenishment decisions. In the case of imperfect observation of knowledge about the environment, healthcare inventory managers or pharmacists follow certain inventory policies that may be different from the case when perfect knowledge is observed. A POMDP-based model, in this case, attempts to consider such situations with which the inventory policies based on the actual environment with absolute certainty may be compared (Arifoğ lu and Özekici 2010a).In this study, details of determining the optimal inventory replenishment policies for stochastic medicine demand is considered, and is modulated by a partially observed random environment together with all cost parameters are presented. The state-dependent optimal ordering policy is computed when the demand process is not perfectly observable. The decisions of the inventory manager are affected by the imperfect observations on the state of this process.

The organization of the paper is as follows. Section-2 reviews the related literature. Section-3 describes the step-wise integrated framework and methodology. The proposed methodology is validated through a case study in Section-4. Section-5 discusses the results. The conclusions and possible future research avenues are discussed in the last section.

2. Related Literature

Existing literature addresses four specific issues, viz. factors influencing prescription behaviour dynamics in physician prescription behaviour, role of physician in inventory management, and inventory management in partially-observed environment. A review of these issues is briefly presented below in **Table-1**.

Table 1 *Related Literature*

Problem Aspect	Modelling Approach	References	Findings
Factors Influencing Prescription Behaviour	Empirical Modelling (Regression	Labi et al. 2018; Pedan & Wu 2011; Lin et al. 2018; Weng et al. 2013; Bhaskarabhatla & Chatterjee 2017; Gourgoulis et al. 2013	Influencing Factors: • Pharmaceutical Detailing and Sampling
Dynamics in Prescription Behaviour	Learning Model; Hidden Markov Model (HMM)	Akçura & Ozdemir 2014; Montoya et al., 2010; Janakiraman et al., 2008;	• Influence of marketing activities • States: Inactive, Infrequent,
Role of physician in healthcare inventory management	Empirical Modelling (Regression Analysis)	Abdulsalam et al., 2018; Nyaga et al., 2015	Recommended standardized practices
Inventory management in Partially-observed	HMM; POMDP	Wang et al., 2010; Bayraktar & Ludkovski, 2010; Treharne & Sox, 2002; Arifoğ lu, K. & Özekici, S., 2010, 2011 Bensoussan et al.,	Partial observations effects optimal inventory policy parameters

3. Methodology

In order to address the research issues and problems identified from the review of literature, a comprehensive methodology is essential. The research methodology for solving the problem as described consists of a number of inter-related steps as shown in Figure-1. There are two parts: (i) To capture physician dynamics in prescription behaviour by hidden Markov model (HMM) and (ii) To determine optimal dynamic inventory replenishment policy by Partially observable Markov decision process (POMDP) using HMM variants. These steps are as discussed below.

Step-1: Represent prescription behaviour as hidden states

There are a finite set of physician prescription-behaviour states. Physicians prescribing fewer dose of a particular medicine may be denoted as the lower prescription states compared to the physicians at the higher prescription state. However, the pharmacists or inventory managers do not have the complete information on the actual state of the physician prescription behaviour. Hence, these states, denoted by *PB* are not observable and are hidden for the pharmacists.

Step-2: Represent the quantity of medicines prescribed as observation process.

The pharmacists cannot observe the hidden process, instead they may only observe a process, for example, quantity of a particular medicine prescribed by physician to a patient for a particular medical condition, denoted by ***OB***. The stock of medicines is maintained based on the quantity of medicines prescribed by the physicians to the patients. The pharmacists or inventory managers may observe the quantity of medicines prescribed which provides partial information about the actual state of prescription and accordingly decide the medicine inventory replenishment policy.

Step-3: Compute the three HMM components.

1. **Initial State Probabilities:** The initial state probabilities denote the probability that the physician, i is initially in state, PB with probability, $P(PB_{i0} = PB) = \pi_{iPB}$.
2. **Markov Chain Transition Matrix**: With time, a physician may change from one prescription state to another due to various factors, such as physician-patient interactions, physician-healthcare facility interactions and physician-pharmaceutical company interactions.
3. **Conditional Probabilities:** The probability that the physician will prescribe the medicine at time t conditioned on the state is denoted as $P(D_{it} = DEM|PB_{it} = PB) = \boldsymbol{P^{OB}}$, where PB_{it} is the state of physician i at time t in a Markov process with Y states, and D_{it} is the dose of medicines prescribed by the physician i at time t. Conditional on being in state PB at time t, the dose of a medicine prescribed by physician, i, follows a negative binomial distribution with parameters r_{it} and p_{iPBt}. It is observed that negative binomial distribution may easily handle such extreme values of zero in the prescription data.

Step-4: Estimate non-homogeneous HMM model parameters.

The non-homogeneous HMM model parameters are estimated using the Baum-Welch algorithm. The algorithm starts by setting the model parameters to some initial values that can be chosen from some prior knowledge. Then, using the current model, all possible paths for each training set are considered to get new estimates. The procedure is repeated until there are insignificant changes in the parameters of the current model.

Step-5: Validate the model using real-life data set collected from a hospital.

The HMM model parameters are validated using the real-time data set collected from a hospital. The data is best represented by a three-state HMM. The parameter estimates are then used to interpret them. To characterize the three states, the medication demand distribution parameters are converted into prescription probabilities conditional on being in each state.

Step-6: Formulate dynamic inventory control problem as POMDP

The inventory system is described by two states (i) inventory level (*INV*) and (ii) physician prescription behaviour. The physician prescription behaviour state is hidden or partially observed through the observation process. The inventory level state is completely observable. Both the state forms time-dependent Markov chains on discrete state spaces. The inventory control problem is formulated as POMDP which is well-suited for handling problems of HMMs. The six elements of POMDP is defined as the states (inventory level and physician prescription behaviour), actions (order quantity), finite set of observations (dose of a medicine prescribed), state transition probabilities (physicians' prescription behaviour state transition and inventory level state transitions), set of observation probabilities and cost incurred function.

Step-7: Define beliefs about physicians' hidden state.

The belief that physician is in the state, PB at the time, t is defined as $b_{it}(PB)$ and the belief state vector is represented as $\boldsymbol{B}_t = \left(b_t(PB = 1), \dots, b_t(PB = Y)\right)'$.

Step-8: Update beliefs about the physicians' state from period t to $t + 1$ after observing physician's decision, using Bayes' rule, transition probability estimates, and conditional probabilities.

$$b_{t+1}(PB|\boldsymbol{B}_t, OB_t) = \frac{\sum_{PB'=1}^{Y} b_{it}(PB') y_{PBPB't} z_{OBPBt}}{\sum_{PB'=1}^{Y} \sum_{l=1}^{Y} b_t(PB') y_{PB'lt} z_{OBlt}} \quad (1)$$

where, $\sum_{PB=1}^{Y} b_t(PB) = 1$ (2)

Step-9: Model the medicine inventory control system as a Dynamic Programming (DP) problem. The objective is to determine, for each period, the optimal inventory policy so as to minimize the sum of expected future costs over a finite planning horizon.

$$\min_{Q_t \geq 0} E\left\{\sum_{t=1}^{T-1} \alpha C_t\right\} \quad (3)$$

where, $\alpha \in [0,1]$ is the discount rate and $E(C_t) = \sum_{PB=1}^{Y} b_t(PB) c_{PBt}$ is the expected cost incurred at period t if physician is in the state PB. Let $TC(\boldsymbol{B}_t, PB, INV, Q)$ denote the value function of the dynamic program associated with belief, $\boldsymbol{B}_t$ at time t = 0, 1,…,T. At time t = 0,

$$TC_0(\boldsymbol{B}, PB, INV, Q) = \min_{Q_t \geq 0} \sum_{PB \in \mathbb{H}} b_0(PB)\ .\ x_{pq}(Q).c_0\,(PB, INV, Q)$$

(4)

where, the transition probabilities of inventory level state, $\boldsymbol{P^{INV}} = \|x_{pq}(Q)\|$ is computed as

$$\boldsymbol{P^{INV}} = \|x_{pq}(Q)\| = Prob(INV_{t+1} = q|INV_t = p, Q_t = Q) = \begin{cases} x_{p+Q-q} & q \leq p+Q \\ \sum_{DEM=p+Q}^{\infty} x_d & q = 0 \\ 0 & q > p+Q \end{cases}$$

(5)

and the expected cost incurred at period *t* if physician state is *PB* and inventory level state is *INV* is calculated as

$$c_0(PB, INV, Q) = K.\delta(Q) + w_r(Q) + w_h \sum_{DEM=0}^{INV+Q-1} (INV + Q - DEM) M_{PB}(DEM) + w_e \sum_{DEM=INV+Q}^{DEM\_Max} (INV + Q - DEM) M_{PB}(DEM)$$

(6)

where, *K* is the fixed order cost per order, *Q* is the order quantity, w_h is the holding cost per period per unit, *DEM* is the demand for medicines, w_e is the expediting or emergency order cost per unit.

At time *t* + 1,

$$TC_{t+1}(\boldsymbol{B}, PB, INV, Q) = \min_{Q_t \geq 0}\left[\sum_{PB \in \mathbb{H}} b_0(PB)\, .\, Prob(INV_{t+1}|INV_t, Q_t).\, c_0(PB, INV, Q) + \alpha \sum_{PB=1}^{Y} b_t(PB).\, Prob(INV_{t+1}|INV_t, Q_t).\, [TC_{t+1}(\boldsymbol{B}_{t+1}, INV_{t+1})]\right]$$

(7)

Step-10: Solve the problem using forward recursive algorithm, and obtain the optimal inventory replenishment policy.
The beliefs, $\boldsymbol{B_t}$ of the hidden states are unknown prior to time, *t* and is available just before the decisions at time, *t* have to be made, therefore, the forward recursive algorithm is applied to solve the inventory control problem (Bertsekas 2005). The algorithm consists of the following steps.

(i) Set the time period as t = 0, and calculate $TC_0(\boldsymbol{B}, PB, INV, Q)$.

(ii) Substitute t + 1 for t and compute $TC_{t+1}(\boldsymbol{B}, PB, INV, Q)$ by satisfying Bellman optimality equation and given the current beliefs.

$$TC(\boldsymbol{B}_t, PB_t, INV_t, Q) = \min_{Q_t \geq 0}\left[E\left\{\sum_{t=0}^{T-1} C_t(\boldsymbol{B}_t, INV, Q)\right\}\right] = \min_{Q_t \geq 0}\left[\sum_{PB \in \mathbb{H}} b_t(PB).\, c_t(PB, INV, Q) + \alpha \sum_{PB=1}^{Y} b_t(PB) E[TC(\boldsymbol{B}_{t+1}, INV_{t+1})]\right]$$

(8)

$$c_T(INV_T) = w_u INV_T$$

(9)

(iii) Set

$$Q(\boldsymbol{B}_t, PB, INV_t) = \arg\min_{Q_t \geq 0}\left[\sum_{PB \in \mathbb{H}} b_t(PB).\, c_t(PB, INV, Q) + \alpha \sum_{PB=1}^{Y} b_t(PB) E[TC(\boldsymbol{B}_{t+1}, INV_{t+1})]\right]$$

(10)

(iv) If t = T – 1, stop; otherwise, return to Step (iii).

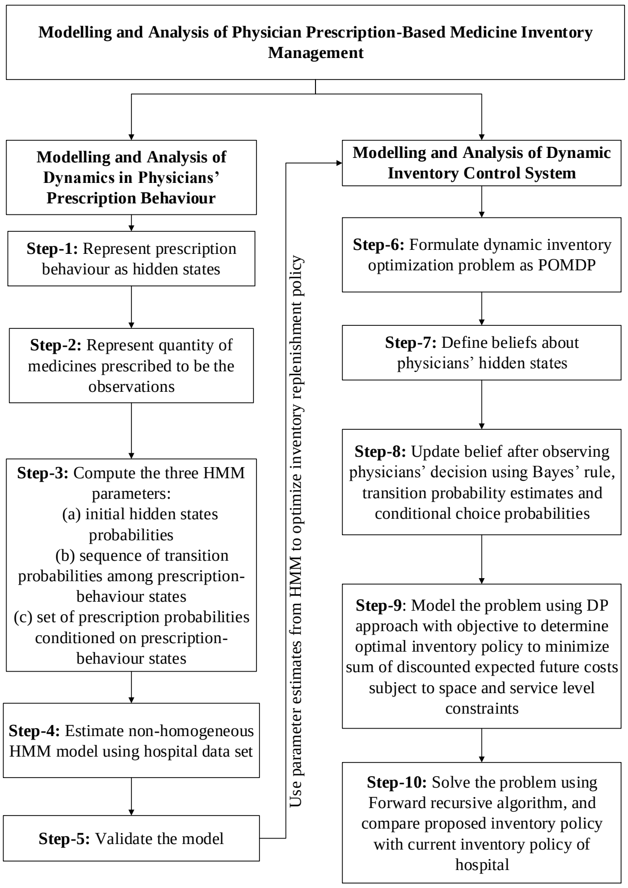


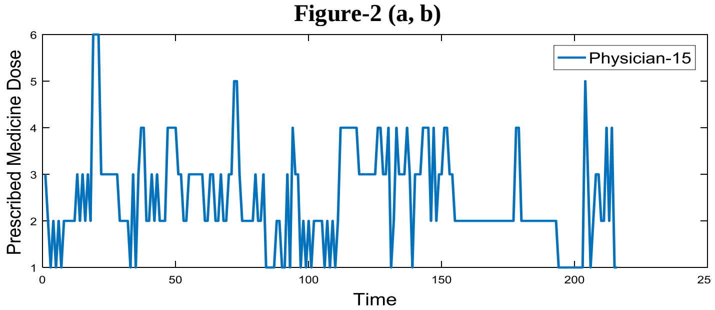
Figure 1 *Integrated Framework*

4. Case Study

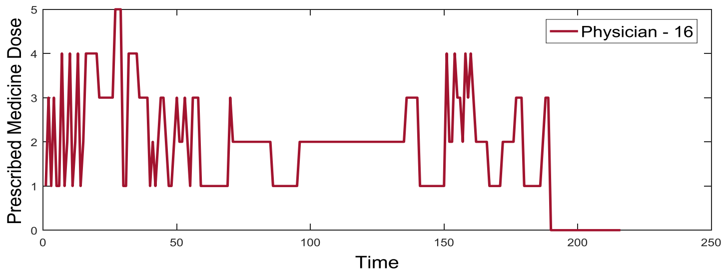
The data used to calibrate and validate the model are sampled from the database provided by a multi-specialty hospital in the eastern part of India. The features of the hospital selected for data collection comprises 15 types of departments, 24-bed types, 226 functional beds, 12000 patient admissions per year, 506 doctors and 5359 types of pharmaceutical products. Three different types of information are collected: (i) Patient-related information, (ii) Physician-related information and (iii) Medicine-related information. The patient-related information includesPatient ID and name, Date of arrival, transferral and discharge, Provisional and final diagnosis and treatment, Type of care unit and Insurance Scheme. Physician-related information includes Patient ID and name, Physician name allotted to patients, their Specialization and Prescription date. The Medicine-related information includes Patient ID and name, Physician name, Name of medicines ordered, Date of purchasing and receiving of order, Quantity ordered and Purchase price of the medicines.

One particular medicine is selected to validate the usability of the model. The generic name of the medicine is Piperacillin – Tazobactam and brand name is Zosyn. Its therapeutic use is for bacterial infections. In aggregate, there are 66 physicians prescribed this medicine in a year. The variability in physician prescription behaviour of two physicians is shown in

Figure-2 (a, b)



(a)



(b)

Figure 2 (a-b) *Variability in Physician Prescription Behaviour*

The demand in high, moderate and low state follows a negative binomial distribution with parameters (r1, p1), (r2, p2) and (r3, p3), respectively as shown in **Table-2**.

Table 2 *Parameters of Demand Distribution*

State 1				**State 2**				**State 3**			
r1	**p1**	**µ1**	**σ1**	**r2**	**p2**	**µ2**	**σ2**	**r3**	**p3**	**µ3**	**σ3**
82.95	0.92	6.44	2.64	126.21	0.88	16.58	4.31	15.22	0.28	39.10	11.73

The inventory-related cost parameter values are based on the real costs incurred at the hospital being studied. The purchase price of the medicine under study is Rs 1027. The unit holding cost is 0.10, i.e. holding a medicine in stock for 1 year costs 36.5, which is 3.5% of its purchase price, which is followed by the hospital administration. The unit outdating cost is decided to be around 20% of purchase price. Because expensive medicines are considered, the expediting cost is considered smaller than the outdating cost. The unit expediting cost is thus around 10% of the purchase price. Without loss of generality, the unit variable ordering cost is set at 0.01 and fixed ordering cost is set at 10. The cost parameters are the same when the observed environment is in any state as the effect due to differences in cost parameters may suppress the effect of the observation level. A three-state HMM is used to describe the physician prescription behaviour of this hospital as described in **Table-3**. The initial state transition matrix is given by the prior distribution. The physicians' prescription behaviour changes after monitoring the patient the next day due to factors like a change in patient condition, type of care unit, etc. Hence, it is assumed that the duration time for each state is one day. This means that the prescription behaviour state may change to other states on the next day or may remain same.

Table 3 *Interpretation of States*

States	**Description**
LOW	Physicians prescribe less dose of medicine to patients
MEDIUM	Physicians prescribe a medium dose of medicine to patients
HIGH	Physicians prescribe a high dose of medicine to patients

Table-4 lists the parameter values used to generate the cases for the computational results. There are 6 cases (1 prior distribution × 1 transition matrix × 6 emission matrices). The parameter values of prior distributions at time 0 indicates that a physician is initially at the low prescription state (state-1). The transition probability matrices model a stable process with positive correlation, which indicates that a physician with time has the propensity to remain in its initial state. The emission probability matrices describe the level of information and are parameterized such that E1 corresponds to the imperfect situation where the observations do not provide complete or perfect information about the real environment. As it moves from E1 to E6 the perfection in the observations improves. Hence, E6 corresponds to the perfect situation where the observations provide complete or perfect information about the real environment.

Table 4 *Model Parameter Values*

Periods	4
Holding Cost, w_h	0.10
Expediting Cost, w_e	100
Ordering Cost: **Fixed** **Variable**	 10 0.01
Outdating Cost	200

Prize Distribution			
	State-1	**State-2**	**State-3**
Initial	1.00	0.00	0.00
H: High	0.10	0.30	0.60
L: Low	0.60	0.30	0.10
Transition Matrices			
SP: Stable Process with Positive Correlation	0.8	0.1	0.1
	0.1	0.8	0.1
	0.1	0.1	0.8
SN: Stable Process with Negative Correlation	0.2	0.4	0.4
	0.4	0.2	0.4
	0.4	0.4	0.2
SZ: Stable Process with Zero Correlation	0.333	0.333	0.333
	0.333	0.333	0.333
	0.333	0.333	0.333
US: Slow Upward Trend Process	0.8	0.1	0.1
	0.0	0.8	0.2
	0.0	0.0	1.0
UF: Fast Upward Trend Process	0.2	0.4	0.4
	0.0	0.2	0.8
	0.0	0.0	1.0
DS: Slow Downward Trend Process	1.0	0.0	0.0
	0.2	0.8	0.0
	0.1	0.1	0.8
DF: Fast Downward Trend Process	1.00	0.0	0.0
	0.8	0.2	0.0
	0.4	0.4	0.2
Emission Probability Matrices			
E1	0.50	0.25	0.25
	0.25	0.50	0.25
	0.25	0.25	0.50
E2	0.60	0.20	0.20
	0.20	0.60	0.20
	0.20	0.20	0.60
E3	0.70	0.15	0.15
	0.15	0.70	0.15
	0.15	0.15	0.70
E4	0.80	0.10	0.10
	0.10	0.80	0.10
	0.10	0.10	0.80
E5	0.90	0.05	0.05
	0.05	0.90	0.05
	0.05	0.05	0.90
E6	1.00	0.00	0.00
	0.00	1.00	0.00
	0.00	0.00	1.00

So far, the parameters of the HMM model are initialized. The state sequence and the optimal model parameters will be continuously updated once new observable prescription behaviour-related information is added. The case with initial state probability, initial transition matrix and imperfect information emission matrix are used as initial guess for illustration purpose, and are then trained accordingly by using the '*hmmtrain*' function in MATLAB. The '*hmmtrain*' function estimates the transition and emission probabilities for a HMM using the Baum-Welch algorithm. The results of the trained HMM model is given below.

$$P_{estimated}^{PB}\begin{bmatrix}0.853 & 0.099 & 0.048\\0.168 & 0.756 & 0.076\\0.194 & 0.122 & 0.684\end{bmatrix}$$

$$P_{estimated}^{OB}\begin{bmatrix}0.628 & 0.001 & 0.371\\0.146 & 0.658 & 0.194\\0.001 & 0.221 & 0.778\end{bmatrix}$$

The belief about the physicians' state is updated using the estimated values of transition matrix and emission matrix and is provided in **Table-5**.

Table5 *Belief State Updating after Actual Observation*

Hidden State	Belief at t = 0	Observed State	Updated Belief at t = 1
1	1		0.8889
2	0	1	0.0556
3	0		0.0556
1	1		0.7273
2	0	2	0.1818
3	0		0.0909
1	1		0.7273
2	0	3	0.0909
3	0		0.1818

5. Results and Discussions

For the numerical illustration, MATLAB is used to code the dynamic programming algorithm for multi-period model. Although it is possible to run the code for N > 4, we prefer N = 4 since this is sufficient to capture the managerial insights. The demand in high, moderate and low state follows negative binomial distribution. The case with initial prior distribution, stable process with positive correlation transition matrix and all observation levels are used as initial guess for illustration purpose and are then trained accordingly. The optimal inventory policy parameters (*s*, *S*) for time 0 to 4 are obtained by solving the optimization problem using the Forward recursion algorithm and the results are provided in **Table-6**which summarizes the optimal threshold levels '*s*' and '*S*' for time 0 to 4 for each case. It is known that '*s*' is the reorder level and '*S*' is the order-up-to level. For example, when the observation level is E_1, optimal *s* and *S* at time 0 is (10, 11) respectively. At time 1, they increase to (11, 15) respectively. From the results at time 1, it is seen that policy parameters are non-decreasing in observation level. When the environment is completely observable with perfect information (E_6), the optimal threshold levels are s = 11 and S = 16 at time t = 1.

The threshold levels in completely unobservable (E1) case are lower compared to the fully observable case (E6). This is intuitive as using the same policy as in the full observation case may lead to tremendous losses if the observed state is not the real one. Moreover, it is observed that the probability of being in high state when it is observed at time 1 is greater than the probability of being in the high state when the low state is observed at time 1. This implies that as the observation level increases, the optimal policy in imperfect information case will converge to the optimal policy in perfect information case. Therefore, it is intuitive that as observation level increases, the threshold level at time t increases if the high state is observed while they decrease if the low state is observed.

6. Conclusions

In this paper, the main aim is to characterize the optimal policy structures for inventory models with different random scenarios in a partially observed random environment. As in the model that consider random environment, the real environment is assumed to follow Markov chain. However, it is also assumed that environmental state is not fully observed. Instead, another process, not necessarily a Markov chain, which gives partial information about the real environment, is observed. Under this setting, the model is analyzed by using sufficient statistic formulation. The model shows that state-dependent (*s, S*) policy is optimal for this type of inventory models with fixed ordering cost.

The future scope of research may be extending the model to obtain optimal policy structure for inventory models with fixed-ordering cost, fully observed random capacity and partially observed availability.

Table-10 *Optimal Inventory Policy*

Case-1: SP

Time Period	E1		E2		E3		E4		E5		E6	
	b1	**(s, S)**	**b2**	**(s, S)**	**b3**	**(s, S)**	**b4**	**(s, S)**	**b5**	**(s, S)**	**b6**	**(s, S)**
t = 0	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)
t = 1	0.2721	(11, 15)	0.5286	(11, 15)	0.6356	(11, 15)	0.7494	(11, 16)	0.8706	(11, 16)	1.0000	(11, 16)
t = 2	0.3685	(11, 21)	0.7936	(11, 21)	0.8907	(11, 22)	0.9516	(11, 22)	0.9852	(11, 22)	1.0000	(11, 22)
t = 3	0.4172	(11, 22)	0.8882	(11, 22)	0.9467	(11, 23)	0.9755	(11, 23)	0.9906	(11, 24)	1.0000	(11, 24)
t = 4	0.4455	(11, 22)	0.9181	(11, 22)	0.9577	(11, 23)	0.9781	(11, 23)	0.9909	(11, 24)	1.0000	(11, 24)

Case-2: SN

Time Period	E1		E2		E3		E4		E5		E6	
	b1	**(s, S)**	**b2**	**(s, S)**	**b3**	**(s, S)**	**b4**	**(s, S)**	**b5**	**(s, S)**	**b6**	**(s, S)**
t = 0	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)
t = 1	0.4870	(11, 16)	0.7353	(11, 17)	0.8121	(11, 17)	0.8810	(11, 17)	0.9434	(11, 18)	1.0000	(11, 18)
t = 2	0.4203	(11, 21)	0.6222	(11, 19)	0.7007	(11, 19)	0.7869	(11, 19)	0.8849	(11, 18)	1.0000	(11, 18)
t = 3	0.4327	(11, 22)	0.6502	(11, 19)	0.7263	(11, 19)	0.8049	(11, 19)	0.8920	(11, 18)	1.0000	(11, 18)
t = 4	0.4298	(11, 22)	0.6432	(11, 19)	0.7205	(11, 19)	0.8016	(11, 19)	0.8912	(11, 18)	1.0000	(11, 18)

Case-3: SZ

Time Period	E1		E2		E3		E4		E5		E6	
	b1	**(s, S)**	**b2**	**(s, S)**	**b3**	**(s, S)**	**b4**	**(s, S)**	**b5**	**(s, S)**	**b6**	**(s, S)**
t = 0	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)
t = 1	0.4423	(11, 16)	0.7041	(11, 17)	0.7873	(11, 17)	0.8638	(11, 17)	0.9345	(11, 17)	1.0000	(11, 17)
t = 2	0.4423	(11, 21)	0.7041	(11, 20)	0.7873	(11, 20)	0.8638	(11, 20)	0.9345	(11, 20)	1.0000	(11, 19)
t = 3	0.4423	(11, 22)	0.7041	(11, 19)	0.7873	(11, 19)	0.8638	(11, 19)	0.9345	(11, 19)	1.0000	(11, 19)
t = 4	0.4423	(11, 22)	0.7041	(11, 19)	0.7873	(11, 19)	0.8638	(11, 19)	0.9345	(11, 19)	1.0000	(11, 19)

Case-4: US

Time Period	E1		E2		E3		E4		E5		E6	
	b1	**(s, S)**	**b2**	**(s, S)**	**b3**	**(s, S)**	**b4**	**(s, S)**	**b5**	**(s, S)**	**b6**	**(s, S)**
t = 0	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)
t = 1	0.1530	(11, 15)	0.3514	(11, 15)	0.4573	(11, 15)	0.5909	(11, 16)	0.7647	(11, 16)	1.0000	(11, 16)
t = 2	0.1918	(11, 20)	0.6653	(11, 21)	0.8197	(11, 22)	0.9233	(11, 22)	0.9792	(11, 22)	1.0000	(11, 22)
t = 3	0.2126	(11, 22)	0.8375	(11, 22)	0.9314	(11, 23)	0.9723	(11, 23)	0.9903	(11, 24)	1.0000	(11, 24)
t = 4	0.2221	(11, 22)	0.9004	(11, 22)	0.9543	(11, 23)	0.9776	(11, 23)	0.9909	(11, 24)	1.0000	(11, 24)

Case-5: UF

Time Period	E1		E2		E3		E4		E5		E6	
	b1	**(s, S)**	**b2**	**(s, S)**	**b3**	**(s, S)**	**b4**	**(s, S)**	**b5**	**(s, S)**	**b6**	**(s, S)**
t = 0	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)
t = 1	0.0522	(11, 14)	0.1417	(11, 16)	0.2043	(11, 16)	0.3056	(11, 16)	0.4976	(11, 17)	1.0000	(11, 18)
t = 2	0.0292	(11, 18)	0.1901	(11, 19)	0.3386	(11, 19)	0.5568	(11, 19)	0.8097	(11, 18)	1.0000	(11, 18)
t = 3	0.0171	(11, 22)	0.2352	(11, 19)	0.4417	(11, 19)	0.6703	(11, 19)	0.8589	(11, 18)	1.0000	(11, 18)
t = 4	0.0104	(11, 22)	0.2697	(11, 19)	0.4432	(11, 19)	0.6987	(11, 19)	0.8635	(11, 18)	1.0000	(11, 18)

Case-6: DS

Time Period	E1		E2		E3		E4		E5		E6	
	b1	**(s, S)**	**b2**	**(s, S)**	**b3**	**(s, S)**	**b4**	**(s, S)**	**b5**	**(s, S)**	**b6**	**(s, S)**
t = 0	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)	0.1000	(10, 11)
t = 1	0.3369	(11, 15)	0.6038	(11, 16)	0.7033	(11, 16)	0.8025	(11, 15)	0.9014	(11, 15)	1.0000	(11, 15)
t = 2	0.5204	(11, 21)	0.8908	(11, 22)	0.9501	(11, 22)	0.9820	(11, 22)	0.9963	(11, 22)	1.0000	(11, 22)

t = 3	0.6421	(11, 22)	0.9725	(11, 23)	0.9922	(11, 24)	0.9984	(11, 24)	0.9999	(11, 25)	1.0000	(11, 25)
t = 4	0.7274	(11, 22)	0.9930	(11, 23)	0.9987	(11, 24)	0.9999	(11, 24)	1.0000	(11, 24)	1.0000	(11, 25)

Case-7: DF

Time Period	**E1**		**E2**		**E3**		**E4**		**E5**		**E6**	
	b1	**(s, S)**	**b2**	**(s, S)**	**b3**	**(s, S)**	**b4**	**(s, S)**	**b5**	**(s, S)**	**b6**	**(s, S)**
t = 0	0.1000	(10,11)	0.1000	(10,11)	0.1000	(10,11)	0.1000	(10,11)	0.1000	(10,11)	0.1000	(10,11)
t = 1	0.6419	(11,16)	0.8432	(11,16)	0.8932	(11,16)	0.9348	(11,16)	0.9699	(11,15)	1.0000	(11,15)
t = 2	0.9206	(11,22)	0.9881	(11,22)	0.9948	(11,22)	0.9981	(11,22)	0.9996	(11,22)	1.0000	(11,22)
t = 3	0.9841	(11,22)	0.9992	(11,23)	0.9997	(11,24)	0.9999	(11,24)	0.9999	(12,25)	1.0000	(12,25)
t = 4	0.9969	(11,22)	0.9992	(11,23)	0.9999	(11,24)	0.9999	(11,24)	0.9999	(12,24)	1.0000	(12,25)

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