A Two-Warehouse Inventory Model with Controllable Deterioration Rate and Time Dependent Quadratic Demand Rate

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This paper presents a two-warehouse inventory model for deteriorating items when the deterioration rate is controllable using preservation technology and the demand rate follows a non-linear trend (i.e. Quadratic form) involving different deterioration rates under permissible delay in payment. The optimal cost of the system is calculated allowing shortages. Further it is assumed that the holding cost is a linear function of time. A numerical example is given and the robustness of the model is tested through sensitive analysis.

Keywords: Quadratic Demand, Deterioration, Preservation Technology, EOQ, Variable Holding Cost

1. Introduction

In a globalised economy one may notice that a fierce competition among wholesalers/suppliers to promote their businesses. In this context they offer a variety of facilities to retailers. Credit on goods is an important facility given by wholesalers/suppliers to sell more goods. A higher rate of interest is charged beyond a particular period of time under some terms and conditions. Haley and Higgins (1973) studied the relationship between inventory policy and credit policy in the context of the classical lot size model. Chapman et al. (1984) developed an economic order quantity model which considers possible credit periods allowable by suppliers. This model is shown to be very sensitive to the length of the permissible credit period and to the relationship between the credit period and inventory level. Davis and Gaither (1985) developed optimal order quantities for firms that are offered a onetime opportunity to delay payment for an order of a commodity. A mathematical model is developed by Goyal (1985) when supplier announces credit period in settling the account, so that no interest charges are payable from the outstanding amount if the account is settled within the allowable delay period. Aggarwal and Jaggi (1995) developed mathematical model for deteriorating inventories for which supplier allowed certain fixed period to settle the account. Shah et al. (1988) extended the above model by allowing shortages. Mandal and Phaujdar (1989a), (1989b) have studied Goyal (1985) model by including interest earned from the sales revenue on the stock remaining beyond the settlement period. Carlson and Rousseau (1989) examined EOQ under date terms supplier credit by partitioning carrying cost into financial cost and variable holding costs. Chung and Huang (2003) extended Goyal (1985) model when replenishment rate is finite. Dallenbach (1986), (1988), Ward and Chapman (1987), Chapman and Ward (1988) argued that the usual assumptions as to the incidence and the value of the inventory investment opportunity cost made by the traditional inventory theory are correct and also established that if trade credit surplus is taken into account, the optimal ordering quantities decreases rather than increase. Chung (1998) established the convexity of the total annual variable cost function for optimal economic order quantity under conditions of permissible delay in payments. Jamal et al. (2000) discussed the problem in which the retailer can pay the supplier either at the end of credit period or later incurring interest charges on the unpaid balance for the overdue period. Sarker et al. (2001) obtained optimal payment time under permissible delay in payments when units in an inventory are subject to deterioration. Abad and Jaggi (2003) considered the seller-buyer channel in which the end demand is price sensitive and the suppler offers trade credit to the buyer. Shinn and Hwang (2003) dealt with the problem of determining the retailer's optimal price and order size simultaneously under the condition of order size dependent delay in payments. It is assumed that the length of the credit period is a function of the retailer's order size and also the demand rate is a function of the selling price. Chung et al. (2005) determined the economic order quantity under conditions of permissible delay in payments where the delay in payments depends on the quantity ordered when the order quantity is less than the quantity at which the delay in payments is permitted, the payment for the item must be made immediately. Otherwise, the fixed credit period is allowed. Huang (2007) examined optimal retailer's replenishment decisions in the EOQ model under two levels of trade credit policy by assuming that the supplier would offer the retailer partially permissible delay in payments when the order quantity is smaller than a predetermined quantity. Tenget. al. (2007) derived retailer's optimal ordering policies with trade credit financing. Venkateswarlu and Reddy (2014) studied inventory models when the demand is time dependent quadratic demand and the delay in payments is acceptable. Venkateswarlu and Reddy (2017) developed a deterministic inventory model for perishable items with price sensitive quadratic time dependent demand under trade credit policy.

After procuring large volume of goods on credit, retailers generally look for storing the inventory in warehouses. Every firm in general has its own warehouse (OW) with a limited capacity. When retailers purchase more goods than the capacity of OW, the excess quantity can be stored in a rented warehouse (RW). Here retailers may also use preservation technology to reduce deterioration rate of goods. The rate of deterioration is faster in some products which causes loss to the retailer. The life span of such products can be increased using some preservatives. This rate of deterioration of items can be controlled using some preservation technology which reduces the deterioration rate thereby the retailer may increase the profit. The deterioration rate of items in the literature is viewed as an exogenous variable which is not controllable. In practice, the deterioration rate of products can be controlled and reduced through various efforts such as procedural changes and specialized equipment acquisition. The consideration of preservation technology is important due to rapid social changes and the fact that preservation technology can reduce the deterioration rate significantly. By the efforts of investing in preservation technology we can reduce the deterioration rate. Mishra (2013) studied an inventory model for deteriorating items with controllable deterioration rate for time dependent demand and holding cost. Venkateswarlu and Reddy (2016) discussed a deterministic inventory model for deteriorating items when the deterioration rate is controllable using preservation technology and the demand rate is a quadratic function of time. Hartley (1976) developed the first two warehouse inventory model. Sarma (1983) developed the inventory model which included two levels of storage and the optimum release rule. Sarma (1987) extended his previous model to the case of infinite refilling rate with shortages. Ghosh and Chakrabarty (2009) developed an order level inventory model with two levels of storage for deteriorating items. An EOQ model with two levels of storage was studied by Dave (1988), considering distinct stage production schemes. Several researchers developed inventory models for deteriorating goods. The deterioration of goods is defined as damage, spoilage, and dryness of items like groceries, pictographic film, electronic equipment, etc. Pakkala and Acharya (1992) developed a two warehouse inventory model for deteriorating items with finite replenishment rate and shortages. Benkherouf (1997) developed a two warehouse model with deterioration and continuous release pattern. Lee and Ma (2000) studied an optimal inventory policy for deteriorating items with two warehouse and time dependent demand. Zhou (2003) developed two warehouse inventory models with time varying demand. Yanlai Liang and Fangming Zhou (2011) developed a two warehouse inventory model for deteriorating items under conditionally permissible delay in payment. Sumdara Rajan and Uthaya kumar (2015) studied a two-warehouse inventory model for deteriorating items with permissible delay under exponentially increasing demand. Naresh Kumar et al (2017) developed a two warehouse inventory model for deteriorating item with exponential demand rate and permissible delay in payment.

Thus in this paper, we propose to study the impact of preservation technology and credit policy for developing a two-warehouse inventory model for deteriorating items with time dependent quadratic demand and time dependent holding cost. The cost of the system is calculated and tested for optimality. A numerical example and the sensitivity of the model is given for managerial applications.

2. Assumptions and Notations

In developing the mathematical model of the inventory system for this study, the following assumptions are used.

2.1 Assumptions

1. The replenishment rate is infinite.
2. Lead time is zero.
3. The inventory model deals with single item
4. Deterioration occurs as soon as items are received into inventory
5. There is no replacement or repair of deteriorating items during the period under consideration
6. The Demand rate $D(t)$ at time‘t’ is assumed to be $D(t) = a + bt + ct^2$ Where $a \geq 0, b \neq 0, c \neq 0$ Here ‘a’ is the initial rate of demand, ‘b’ is the initial rate of change of the demand and ‘c’ is the acceleration of demand rate.
7. Shortages are not allowed to occur.
8. The OW has a fixed capacity of W units and the RW has unlimited or infinite capability.
9. The RW is utilized only after OW is full, but stocks in RW are dispatched first.
10. The holding cost is h per unit of time (excluding interest charges), when h = h_o for items in OW and h = h_r for items in RW and $h_r > h_o$.
11. The items deteriorate at a constant rate α in OW and at θ in RW.

2.2 Notations

In developing the mathematical model of the inventory system for this study, the following assumptions are used.

1. **A** is the Ordering cost per order.
2. ***P*** is the unit purchase cost
3. ***s*** is the unit selling price $(s > p)$.
4. Preservation technology (PT) cost is denoted by ξ which reduces the deterioration rate in order to preserve the product, $\xi > 0$.
5. θ is the deterioration rate.
6. $m(\xi)$ is the reduced deterioration rate due to preservation technology.
7. τ_p is the resultant deterioration rate, $\tau_p = \theta - m(\xi)$.
8. $\boldsymbol{h_r}$ is unit stock holding cost per unit of time in rented warehouse (excluding interest charges).
9. $\boldsymbol{h_o}$ is unit stock holding cost per unit of time in owned warehouse (excluding interest charges)

10. ***Q(t)*** is the Ordering quantity at time $t = 0$
11. ***Ie*** is the interest earned per year per unit of time by retailer
12. ***Ic*** is the interest charged per stocks per year per unit of time by supplier.
13. ***w***is the capacity of the owned warehouse (OW)
14. w_1 is the maximum inventory level.
15. t_w is the time that inventory level reduce to W (decision variable)
16. ***M*** is the retailer's trade credit period offered by supplier per year, $0<M<1$.
17. ***T*** is the interval between two successive orders.
18. $I_r(t)$is the inventory level at time $t \in [0, tw]$ in rented warehouse (RW)
19. $I_o(t)$is the inventory level at time $t \in [0, T]$ in owned warehouse (OW)
20. $TC_1(t_w,T,\xi)$, $TC_2(t_w,T,\xi)$ and $TC_3(t_w,T,\xi)$ are the total cost per unit time in a two-warehouse model.

3. Formulation and Solution of the Model

A lot size of particular units enters into the inventory system at time $t = 0$. In OW, *w* units are kept and the remaining units are stored in RW. The items stored in OW are consumed only when the items in RW are consumed first. The stock in RW decreases owing to combined effects of demand and deterioration during the interval [0, t_w] and it vanishes at $t = t_w$. However, the stock in OW depletes due to deterioration only during [0, t_w]. But during [t_w,T], the stock decreases due to combined effects of demand and deterioration. At time T, both the warehouses are empty. The entire process is repeated for every replenishment cycle.

The inventory level in RW and OW at time $t \in [0, t_w]$ is described by following differential equations:

$$\frac{dI_r(t)}{dt} + \tau_p I_r(t) = -D(t), \qquad 0 \le t \le t_w \tag{1}$$

Where $\tau_p = \theta - m(\xi)$, $\theta = kt$ and with boundary conditions $I_r(t) = 0$.

$$\frac{dI_0(t)}{dt} = -\alpha I_0(t), \qquad 0 \le t \le t_w \tag{2}$$

With boundary condition $I_0(0) = w$

$$\frac{dI_0(t)}{dt} + \alpha I_0(t) = -D(t), \qquad t_w < t \le T \tag{3}$$

With boundary condition $I_0(T) = 0$

The solutions of the above system are

$$I_r(t) = \begin{Bmatrix} a(t_w - t) + (b - am(\xi))\left(\frac{t_w^2}{2} - \frac{t^2}{2}\right) + \left(\frac{ka}{2} + c - bm(\xi)\right)\left(\frac{t_w^3}{3} - \frac{t^3}{3}\right) \\ + \left(\frac{kb}{2} - cm(\xi)\right)\left(\frac{t_w^4}{4} - \frac{t^4}{4}\right) + \frac{kc}{2}\left(\frac{t_w^5}{5} - \frac{t^5}{5}\right) \end{Bmatrix} \tag{4}$$
$$- \left(\frac{kt^2}{2} - m(\xi)t\right)\left(a(t_w - t) + b\left(\frac{t_w^2}{2} - \frac{t^2}{2}\right) + c\left(\frac{t_w^3}{3} - \frac{t^3}{3}\right)\right)$$

$$I_0(t) = a(T - t) + (a\alpha + b)\left(\frac{T^2}{2} - \frac{t^2}{2}\right) + (b\alpha + c)\left(\frac{T^3}{3} - \frac{t^3}{3}\right) + c\alpha\left(\frac{T^4}{4} - \frac{t^4}{4}\right)$$
$$- (\alpha t)\left[a(T - t) + b\left(\frac{T^2}{2} - \frac{t^2}{2}\right) + c\left(\frac{T^3}{3} - \frac{t^3}{3}\right)\right] \tag{5}$$

Since the maximum inventory level w_1 is defined as $w_1 = I_r(0) + I_0(0)$, then we have

$$w_1 = at_w + (b - am(\xi))\left(\frac{t_w^2}{2}\right) + \left(\frac{ka}{2} + c - bm(\xi)\right)\left(\frac{t_w^3}{3}\right) + \left(\frac{kb}{2} - cm(\xi)\right)\left(\frac{t_w^4}{4}\right) + \frac{kc}{2}\left(\frac{t_w^5}{5}\right) + w \tag{6}$$

The total relevant costs, TC, comprise following elements:

1. **The ordering cost = A**
2. **Cumulative inventories of stock holding cost during [0,T] is**

$$HC = h_r \int_0^{t_w} I_r(t)\,dt + h_0 \int_0^{t_w} I_0(t)\,dt + h_0 \int_{t_w}^{T} I_0(t)\,dt$$

$$= h_r\left(\frac{at_w^2}{2} + \frac{bt_w^3}{3} + \frac{ct_w^4}{4} + \frac{akt_w^4}{12} + \frac{bkt_w^5}{15} + \frac{5ckt_w^6}{36} - \frac{at_w^3 m(\xi)}{6} - \frac{bt_w^4 m(\xi)}{8} - \frac{ct_w^5 m(\xi)}{10}\right)$$

$$-\frac{h_0 w\left(e^{-\alpha t_w} - 1\right)}{\alpha} + \frac{h_0 (T - t_w)^2}{120}\begin{pmatrix} 60a + 30T^2c + 10ct_w^2 + 40Tb + 20bt_w + 20Ta\alpha \\ -20Tct_w - 20at_w\alpha + 15T^2b\alpha + 12T^3c\alpha - 5bt_w^2\alpha \\ -2ct_w^3\alpha - 10Tbt_w\alpha - 4Tct_w^2\alpha - 6T^2ct_w\alpha \end{pmatrix}$$

3. **The Deteriorating cost [0,T] is**

$$DC = \tau_p \int_0^{t_w} I_r(t)\,dt + \alpha \int_0^{T} I_0(t)\,dt$$

$$= \tau_p \int_0^{t_w} I_r(t)\,dt + \alpha \int_0^{t_w} I_0(t)\,dt + \alpha \int_{t_w}^{T} I_0(t)\,dt$$

$$DC = \tau_p\left(\frac{at_w^2}{2} + \frac{bt_w^3}{3} + \frac{ct_w^4}{4} + \frac{akt_w^4}{12} + \frac{bkt_w^5}{15} + \frac{5ckt_w^6}{36} - \frac{at_w^3 m(\xi)}{6} - \frac{bt_w^4 m(\xi)}{8} - \frac{ct_w^5 m(\xi)}{10}\right)$$

$$-\frac{w\alpha\left(e^{-\alpha t_w} - 1\right)}{\alpha} + \frac{\alpha (T - t_w)^2}{120}\begin{pmatrix} 60a + 30T^2c + 10ct_w^2 + 40Tb + 20bt_w + 20Ta\alpha \\ -20Tct_w - 20at_w\alpha + 15T^2b\alpha + 12T^3c\alpha - 5bt_w^2\alpha \\ -2ct_w^3\alpha - 10Tbt_w\alpha - 4Tct_w^2\alpha - 6T^2ct_w\alpha \end{pmatrix}$$

4. **The interest is payable is calculated as follows:**

Case-1(a): $M \le t_w < T$ Based on the parameters t_w, T and M there are three cases to be considered.

Case (a): In this case, the interest is payable is

$$IP_1 = pI_c \int_M^{t_w} I_r(t)\,dt + pI_c \int_M^{t_w} I_0(t)\,dt + pI_c \int_{t_w}^{T} I_0(t)\,dt$$

$$IP_1 = \frac{pI_c (M - t_w)^2}{360}\begin{pmatrix} 180a + 30M^2c + 90ct_w^2 + 60Mb + 120bt_w - 45bt_w^2 m(\xi) \\ -36t_w^3 m(\xi) + 60Mct_w + 60Mam(\xi) - 30M^2ak - 9M^3bk \\ +2M^4ck - 60at_w m(\xi) + 30akt_w^2 + 24bkt_w^3 + 50ckt_w^4 \\ +15M^2bm(\xi) + 6M^3cm(\xi) + 18Mct_w^2 m(\xi) + 12M^2ct_w m(\xi) \\ +6M^2ckt_w^2 + 30Mbt_w m(\xi) + 3Mbkt_w^2 - 18M^2bkt_w \\ +28Mckt_w^3 + 4M^3ckt_w \end{pmatrix}$$

$$-\frac{pI_c w\left(e^{-\alpha t_w} - e^{-\alpha M}\right)}{\alpha}$$

$$+\left(\frac{pI_c (T - t_w)^2}{120}\right)\begin{pmatrix} 60a + 30T^2c + 10ct_w^2 + 40Tb + 20bt_w + 20Ta\alpha \\ -20Tct_w - 20at_w\alpha + 15T^2b\alpha + 12T^3c\alpha - 5bt_w^2\alpha \\ -2ct_w^3\alpha - 10Tbt_w\alpha - 4Tct_w^2\alpha - 6T^2ct_w\alpha \end{pmatrix}$$

Case-1(b): When $t_w \le M < T$ **,** the interest payable is

$$IP_2 = pI_c \int_M^{T} I_0(t)\,dt$$

$$= \frac{pI_c (M - T)^2}{120} \begin{pmatrix} 60a + 30T^2 c + 10cM^2 + 40Tb + 20bM + 20Ta\alpha \\ -20TcM - 20aM\alpha + 15T^2 b\alpha + 12T^3 c\alpha - 5bM^2\alpha \\ -2cM^3\alpha - 10TbM\alpha - 4TcM^2\alpha - 6T^2 cM\alpha \end{pmatrix}$$

Case-1(c): If $M > T$, the interest payable is zero

5. The interest earned is calculated in the following two cases

Case-1(a): $M < T$
In this case the interest earned is

$$IE_1 = sI_e \int_0^M D(t)t\,dt$$

$$= sI_e \int_0^M \left(a + bt + ct^2\right)(t)dt$$

$$= sI_e \left(\frac{M^2 (3cM^2 + 4bM + 6a)}{12} \right)$$

Case-1(b): $M > T$
In this case the interest earned is

$$IE_2 = sI_e \left(\int_0^T D(t)t\,dt + D(T)T(M - T) \right)$$

$$= sI_e \left(\int_0^T \left(a + bt + ct^2\right)t\,dt + T(M - T)(cT^2 + bT + a) \right)$$

$$= sI_e \left(\frac{T^2 (3cT^2 + 4bT + 6a)}{12} + T(M - T)(cT^2 + bT + a) \right)$$

Thus, the total relevant cost per year for the retailer is given by

$$TC(t_w, T, \xi) = \left(\frac{1}{T}\right) \begin{pmatrix} \text{Ordering cost} + \text{Stock holding cost in RW} \\ + \text{Stock holding cost in OW} + \text{Deterioration cost} \\ + \text{Opportunity cost with interest} - \text{Interest earned} \end{pmatrix} \quad (7)$$

The total relevant costs for the retailer are given as

$$TC(t_w, T, \xi) = \begin{cases} TC_1 & M \le t_w < T \\ TC_2 & t_w \le M < T \\ TC_3 & M > T \end{cases}$$

Where

$$\therefore TC_1=\left(\frac{1}{T}\right)\left[\begin{array}{l} A+\left(h_r+\tau_p\right)\left(\begin{array}{l}\frac{at_w^2}{2}+\frac{bt_w^3}{3}+\frac{ct_w^4}{4}+\frac{akt_w^4}{12}+\frac{bkt_w^5}{15}+\frac{5ckt_w^6}{36}-\frac{at_w^3m(\xi)}{6}\\ -\frac{bt_w^4m(\xi)}{8}-\frac{ct_w^5m(\xi)}{10}\end{array}\right)\\ +\frac{pI_c(M-t_w)^2}{360}\left(\begin{array}{l}180a+30M^2c+90ct_w^2+60Mb+120bt_w-45bt_w^2m(\xi)\\ -36t_w^3m(\xi)+60Mct_w+60Mam(\xi)-30M^2ak-9M^3bk\\ +2M^4ck-60at_wm(\xi)+30akt_w^2+24bkt_w^3+50ckt_w^4\\ +15M^2bm(\xi)+6M^3cm(\xi)+18Mct_w^2m(\xi)+12M^2ct_wm(\xi)\\ +6M^2ckt_w^2+30Mbt_wm(\xi)+3Mbkt_w^2-18M^2bkt_w\\ +28Mckt_w^3+4M^3ckt_w\end{array}\right)\\ -\frac{(h_0+\alpha)w\left(e^{-\alpha t_w}-1\right)}{\alpha}-\frac{pI_cw\left(e^{-\alpha t_w}-e^{-\alpha M}\right)}{\alpha}-sI_e\left(\frac{M^2(3cM^2+4bM+6a)}{12}\right)\\ +\frac{(h_0+\alpha+pI_c)(T-t_w)^2}{120}\left(\begin{array}{l}60a+30T^2c+10ct_w^2+40Tb+20bt_w+20Ta\alpha\\ -20Tct_w-20at_w\alpha+15T^2b\alpha+12T^3c\alpha-5bt_w^2\alpha\\ -2ct_w^3\alpha-10Tbt_w\alpha-4Tct_w^2\alpha-6T^2ct_w\alpha\end{array}\right)\end{array}\right]$$

$$\therefore TC_2=\frac{1}{T}\left[\begin{array}{l} A+\left(h_r+\tau_p\right)\left(\begin{array}{l}\frac{at_w^2}{2}+\frac{bt_w^3}{3}+\frac{ct_w^4}{4}+\frac{akt_w^4}{12}+\frac{bkt_w^5}{15}+\frac{5ckt_w^6}{36}-\frac{at_w^3m(\xi)}{6}\\ -\frac{bt_w^4m(\xi)}{8}-\frac{ct_w^5m(\xi)}{10}\end{array}\right)\\ -\frac{(h_0+\alpha)w\left(e^{-\alpha t_w}-1\right)}{\alpha}-sI_e\left(\frac{M^2(3cM^2+4bM+6a)}{12}\right)\\ +\frac{(h_0+\alpha)(T-t_w)^2}{120}\left(\begin{array}{l}60a+30T^2c+10ct_w^2+40Tb+20bt_w+20Ta\alpha\\ -20Tct_w-20at_w\alpha+15T^2b\alpha+12T^3c\alpha-5bt_w^2\alpha\\ -2ct_w^3\alpha-10Tbt_w\alpha-4Tct_w^2\alpha-6T^2ct_w\alpha\end{array}\right)\\ +\frac{pI_c(M-T)^2}{120}\left(\begin{array}{l}60a+30T^2c+10cM^2+40Tb+20bM+20Ta\alpha\\ -20TcM-20aM\alpha+15T^2b\alpha+12T^3c\alpha-5bM^2\alpha\\ -2cM^3\alpha-10TbM\alpha-4TcM^2\alpha-6T^2cM\alpha\end{array}\right)\end{array}\right]$$

$$\therefore TC_3=\frac{1}{T}\left[\begin{array}{l} A+\left(h_r+\tau_p\right)\left(\begin{array}{l}\frac{at_w^2}{2}+\frac{bt_w^3}{3}+\frac{ct_w^4}{4}+\frac{akt_w^4}{12}+\frac{bkt_w^5}{15}+\frac{5ckt_w^6}{36}-\frac{at_w^3m(\xi)}{6}\\ -\frac{bt_w^4m(\xi)}{8}-\frac{ct_w^5m(\xi)}{10}\end{array}\right)\\ -\frac{(h_0+\alpha)w\left(e^{-\alpha t_w}-1\right)}{\alpha}-sI_e\left(\begin{array}{l}\frac{T^2(3cT^2+4bT+6a)}{12}\\ +T(M-T)(cT^2+bT+a)\end{array}\right)\\ +\frac{(h_0+\alpha)(T-t_w)^2}{120}\left(\begin{array}{l}60a+30T^2c+10ct_w^2+40Tb+20bt_w+20Ta\alpha\\ -20Tct_w-20at_w\alpha+15T^2b\alpha+12T^3c\alpha-5bt_w^2\alpha\\ -2ct_w^3\alpha-10Tbt_w\alpha-4Tct_w^2\alpha-6T^2ct_w\alpha\end{array}\right)\end{array}\right]$$

The optimal values of t_w, T and ξ obtained by solving

$\frac{\delta TC_i(t_w,T,\xi)}{\delta t_w}=0$, $\frac{\delta TC_i(t_w,T,\xi)}{\delta T}=0$ and $\frac{\delta TC_i(t_w,T,\xi)}{\delta\xi}=0$ for i =1, 2, 3

Provided the determinants of principal minor of hessian matrix (H-Matrix) of $TC_i(t_w,T,\xi)$ is positive definite. i.e., det (H_1) >0, det (H_2) >0, det (H_3) > 0, where H_1, H_2, H_3 are the principal minors of the H-Matrix. The Hessian matrix of the total cost $TC_i(t_w,T,\xi)$ is given by

$$H=\begin{bmatrix}\frac{\partial^2 TC}{\partial t_w^2} & \frac{\partial^2 TC}{\partial t_w\partial T} & \frac{\partial^2 TC}{\partial t_w\partial\xi}\\ \frac{\partial^2 TC}{\partial T\partial t_w} & \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T\partial\xi}\\ \frac{\partial^2 TC}{\partial\xi\partial t_w} & \frac{\partial^2 TC}{\partial\xi\partial T} & \frac{\partial^2 TC}{\partial\xi^2}\end{bmatrix}$$

Using these optimal values of t_w, T and ξ the optimal value of w_1 can be obtained from the equation (4)

6. Numerical Example

The following hypothetical data is taken to validate the effectiveness of the models developed

T =1,	a = 287, b = 20,	c = 0.05,	A = 250,	α = 0.1,
ξ = 1.5 ,	h_r = 3,	h_0 = 1,	w = 100, M = 0.15,	I_c = 0.12,
I_e = 0.09,	s = 5,	p = 2,	t_w = 0.05 k = 0.3,	

And $m(\xi) = \theta\left(1 - e^{-2\xi}\right)$

The optimality conditions given by (17) and (18) are satisfied all types of Total costs with the choice of the parameters given above. For these values the optimum values of t_w, cycle time T, total cost TC_1 and the maximum inventory level w_1 of the system are 0.293, 1.236, 440.681 and 114.376 respectively. Table-1 shows the results of various models. It is observed that the value of maximum inventory level w_1 of the system is same and the values of t_w, cycle time (T) and total cost $TC_1$are slightly changed in these models.

Table 1

Model	t_w	T	ξ	$TC(t_w,T)$	w_1
TC_1	0.293	1.236	17.689	440.681	114.376
TC_2	0.248	1.187	4.718	435.982	114.376
TC_3	0.218	1.085	3.953	460.439	114.376

4. Sensitive Analysis

We now study sensitivity of the models developed to examine the implications of underestimating and overestimating the parameters individually on optimal value of total cost. The Sensitive analysis is performed by changing each of the parameter by -15%, -5%, +5% and +15% taking one parameter at a time and keeping the remaining parameters are unchanged. Since all models show slightly variation in results, we will present the sensitivity for total cost for the first case. The results are shown in Table-2. The following observations are made from this Table

- The Total cost function TC_1 is highly sensitive to the changes in the parameter 'a', 'A', 'h_0', 'α', 'M', 'I_c', and 'p'.
- The Total cost function TC_1 is moderately sensitive to the changes in the parameter 'b', 'w', 'Ie', 's',
- The Total cost function TC_1 is less sensitive to all other parameters namely '*c*', 'θ', and 'h_r'.
- The maximum inventory level $w_1$is highly sensitive to the changes in the parameter 'w', 'a' and there is no sensitive to all other parameters 'A', 'h_0', 'α', 'M', 'I_c', 'b', 'w', 'Ie', 's', 'p', '*c*', 'θ', and 'h_r'.

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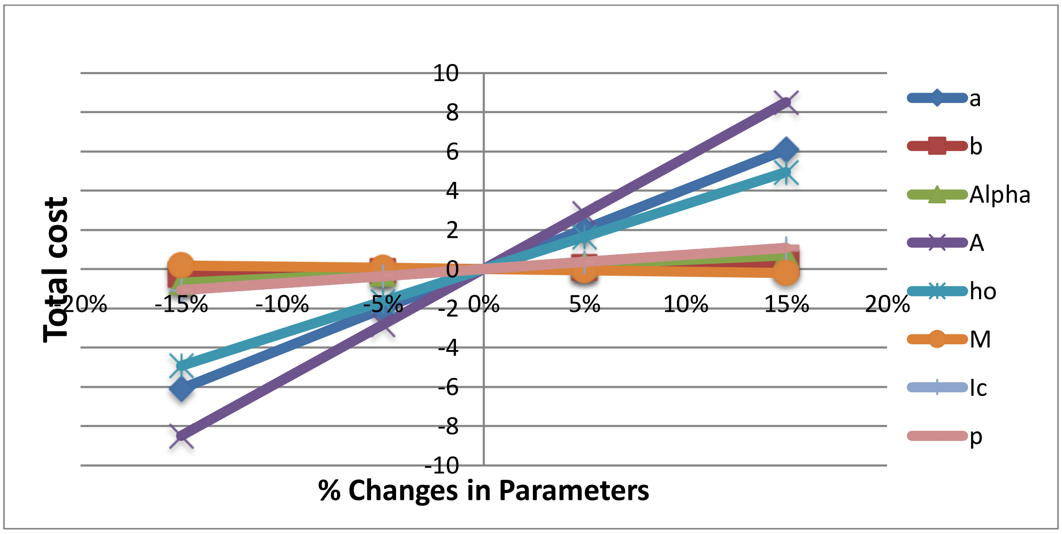

Figure 1 *Variations of Total cost w.r.t the Values of some Important Parameters*

5. Conclusions

1. In this paper we have developed inventory model for two-warehouse having quadratic time dependent demand rate with constant rates of deterioration with trade credit policy by using preservation technology.
2. The main emphasis of this paper is on cost reduction by making effective capital investment in preservation technology.

Table 2

Parameters	%change	$\%t_w$	%T	$\%TC_1$	$\%w_1$
a	-15%	4.437	6.715	-6.093	-1.882
	-5%	1.706	2.104	-2.031	-0.628
	5%	-1.365	-2.023	2.031	0.627
	15%	-3.754	-5.663	6.093	1.882
b	-15%	0.341	0.647	-0.291	-0.003
	-5%	0.341	0.243	-0.097	-0.001
	5%	0.000	-0.243	0.097	0.001
	15%	-0.341	-0.647	0.291	0.003
c	-15%	0.000	0.000	0.000	0.000
	-5%	0.000	0.000	0.000	0.000
	5%	-17.747	-8.010	0.000	0.000
	15%	0.000	0.000	0.001	0.000
A	-15%	-10.239	-7.686	-8.510	0.000
	-5%	-3.413	-2.508	-2.837	0.000
	5%	3.413	2.427	2.837	0.000
	15%	9.898	7.039	8.510	0.000
alpha	-15%	-0.683	0.647	-0.689	0.000
	-5%	0.000	0.243	-0.230	0.000
	5%	0.341	-0.243	0.231	0.000
	15%	0.683	-0.647	0.693	0.000
hr	-15%	13.993	2.265	-0.037	0.000
	-5%	-82.935	-19.094	-0.012	0.000
	5%	-3.754	-0.647	0.012	0.000
	15%	-82.935	-19.094	0.037	0.000
h0	-15%	-2.048	4.207	-4.935	0.000
	-5%	-0.683	1.294	-1.645	0.000
	5%	0.683	-1.214	1.645	0.000
	15%	2.048	-3.560	4.936	0.000
w	-15%	5.461	0.485	-0.106	-13.115
	-5%	2.048	0.162	-0.035	-4.372
	5%	-1.706	-0.162	0.035	4.372
	15%	-5.461	-0.566	0.106	13.115
M	-15%	0.000	0.081	0.182	0.000
	-5%	0.000	0.000	0.061	0.000
	5%	0.341	0.000	-0.062	0.000
	15%	0.341	-0.081	-0.189	0.336
Ic	-15%	-82.935	-19.094	-1.074	0.000
	-5%	0.000	0.405	-0.358	0.000
	5%	-82.935	-19.094	0.358	0.000
	15%	-83.959	-15.210	1.075	0.000
Ie	-15%	0.000	0.000	0.050	0.000
	-5%	0.000	0.000	0.017	0.000
	5%	0.000	0.000	-0.017	0.000
	15%	0.000	-0.081	-0.050	0.000
s	-15%	0.000	0.000	0.050	0.000
	-5%	0.000	0.000	0.017	0.000
	5%	0.000	0.000	-0.017	0.000
	15%	0.000	-0.081	-0.050	0.000
p	-15%	-82.935	-19.094	-1.074	0.000
	-5%	0.000	0.405	-0.358	0.000
	5%	-82.935	-19.094	0.358	0.000
	15%	-83.959	-15.210	1.075	0.000

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