Production Inventory Model with Weibull Deterioration Rate, Time Dependent Quadratic Demand and Variable Holding Cost

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This paper presents production lot size inventory models for deteriorating items with time dependent quadratic demand rate. It is assumed that the deterioration rate follows Weibull distribution. It is further assumed that the holding cost is a linear function of time. Inventory models are developed without considering shortages. The salvage value is considered while calculating the optimal policies that maximize the revenue of the system. Numerical example is given and discussed the sensitivity of these models.

Keywords: Production, Quadratic Demand, Weibull, Holding Cost, Inventory Model.

1. Introduction

Operations Research (OR) addresses the process of decision making in business enterprises and industries. It is known that the inventory management system is one of the important field of study in OR. The study of deteriorating items in inventory system has gained the attention of many researchers in this area of research. The study of the inventory of deteriorating items was opened up by within [1]. In his study, he discussed the deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader [2] extended the classical EOQ formula with exponential decay of inventory due to deterioration and gave a mathematical model of inventory of deteriorating items. The literature is replete with inventory models for deteriorating items on the basis of demand variations and various other conditions or constraints.

One important problem faced in supply chain management in today's context is to control the inventory for deteriorating items. Usually deterioration is defined as the damage, spoilage, pilferage, dryness, vaporization, etc., that result in decrease of usefulness of the original one. It is believed that goods deteriorate over time. The rate of deterioration depends on the type of good. Electronic products may become absolute as technology changes. Fashion goods tend to depreciate the value of clothing over time. The effect of time is even more critical for perishable goods such as food stuffs and cigarettes. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. In realistic terms, the product may be understood to have life time which ends when utility reaches zero. Haiping and Wang [7] developed an economic policy model for deteriorating items with time proportional demand. Donaldson [8] derived an analytical solution to the problems of obtaining the optimal number of replenishments and the optimal replenishment times of an EOQ model with a linearly time dependent demand pattern over a finite time horizon. Zangwill [9] developed a discrete-in-time dynamic programming algorithm to solve an inventory model by allowing the inventory levels to be negative where the demand pattern is time dependent. Following the approach of Donaldson [8], Murdeshwar [6] has tried to derive an exact solution for a finite horizon inventory model to obtain the optimal number of replenishments, optimal replenishments times and the optimal times at which the inventory level falls to Zero, assuming the demand rate to be linearly time dependent and shortages. Hamid [3], Kun-Shan Wu et.al [5] presented a heuristic model for determining the ordering schedule when inventory items are subjects to deterioration and demand changes linearly over time and obtained an optimal replenishment cycle length. Goswami and Chaudhuri [2] presented an EOQ model deteriorating items with shortage and linear trend in demand. Brad Shaw and Erol [1] published a paper in which they derived unbounded control policies for a class of linear time invariant production inventory systems.

All these works were based on the assumption that the demand rate is either linear or exponential function of time. Several researchers argued that, in realistic terms, the demand need not follow either linear or exponential trend. It is well known that the demand for spare parts of new aero planes, computer chips of advanced computer machines, etc. increase very rapidly while the demands for spares of the obsolete aero planes, computers etc. decrease very rapidly with time. This type of phenomena can well be addressed by inventory models with quadratic demand rate [i.e., $D(t) = a + bt + ct^2$; $a \geq 0, b \neq 0, c \neq 0$]. The functional form of time-dependent quadratic demand explains the accelerated (or retarded) growth (or decline) in the demand patterns which may arise due to seasonal demand rate (Khanra and Chaudhuri [4]). One can explain different types of realistic demand patterns depending on the signs of b and c. Bhandari and Sharma [5] have studied a single period inventory problem with quadratic demand distribution under the influence of marketing policies. Khanra and Chaudhuri [4] have discussed an order-level inventory problem with the demand rate represented by a continuous quadratic function of time. Sana and Chaudhuri [6] have developed a stock-review inventory model for perishable items with uniform replenishment rate and stock-dependent demand. Ghosh and Chaudhuri [7] have developed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand and shortages in inventory. They have used a two-parameter Weibull distribution to represent the time to deterioration. Venkateswarlu and Mohan [8] have developed inventory models for deteriorating items with time dependent quadratic demand and salvage value. Venkateswarlu and Mohan [9] studied inventory model for time varying deterioration and price dependent quadratic demand with salvage value. Venkateswarlu and Reddy [10] developed time dependent quadratic demand inventory model under inflation. Venkateswarlu and Reddy [11] studied inventory models when the demand is time dependent quadratic demand and the delay in payments is acceptable. Begum et al [25] developed an EOQ model with shortages for deteriorating items with Weibull distribution and unit production cost with quadratic demand in time. They have further assumed that the production cost is inversely proportional to the demand rate. Kalam et al [26] also developed a lot-size inventory model for deteriorating items with Weibull distribution, quadratic demand and shortages.

Thus in this paper, it is proposed to develop inventory models for deteriorating items which follow Weibull distribution, variable holding cost and time dependent quadratic demand rate. It is further assumed that the salvage value to optimize the total revenue of the system. Numerical example is given to test the robustness of the model. Sensitivity analysis is carried out to determine the most sensitive parameters in the model.

2. Assumptions and Notations

1. The demand rate is assumed to be $D(t). = a + b.t + c.t^2$, where *a, b* and *c* being constants.
2. *I(t)* is the Inventory level at time *t.*
3. The lead-time is Zero and shortages are allowed.
4. Planning horizon is finite.
5. The production rate say $K = \gamma.D(t),$ where $\gamma > 1$.
6. The fraction of the on-hand inventory deteriorates per unit time, where $\theta(t) = \alpha\beta t^{\beta-1}$, $0 < \alpha < 1, t > 1$ *and* $\beta \geq 1$.
7. The production dominates demand and deterioration during the time 0 to t_1, and the Inventory level accumulates.
8. There is no production during the time t_1 to t_2 and demand and deterioration dominate and so the inventory level gradually depletes to zero.
9. Holding cost is linear function of time $h(t) = a_1 + a_2 t, \quad a_1 \geq 0,\ a_2 \geq 0$.
10. b_1 is the Deteriorating cost per unit time.
11. γ_1 is the salvage value, associated with deteriorated units during a cycle time.
12. S_r is the selling price per unit.
13. $T(= t_1 + t_2)$ is the prescribed time period.

3. Formulation and Solution of the Mathematical Model

The objective of the model is to determine the optimum profit for items having time dependent quadratic demand and the rate of deterioration follows Weibull distribution. It is assumed that the production dominates demand and deterioration during the time 0 to t_1. Further it is assumed that there is no production during the time t_1 to t_2 and demand and deterioration dominate, so that the inventory level gradually depletes to zero at the end.

If *I(t)* be the inventory level at time t, the differential equations which describes the inventory level at time *t* are given by

$$\frac{dI(t)}{dt} + \theta(t).I(t) = K - R(t), \quad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dI(t)}{dt} + \theta(t).I(t) = -R(t), \quad t_1 \leq t \leq t \tag{2}$$

Where $K = \gamma.R(t),$ $R(t) = a + b.t + c.t^2$ and $\theta(t) = \alpha\beta t^{\beta-1}$

Our assumptions imply that

$I(0) = 0, I(t_1) = S$ and $I(t_2) = 0$ (3)

The solution of equation (1) with the condition *I(0) = 0* is

$$I(t) = (\gamma - 1).(1 - \alpha.t^{\beta}).\left(at + \frac{b.t^2}{2} + \frac{c.t^3}{3} + (\alpha)\left(\frac{at^{\beta+1}}{\beta+1} + \frac{b.t^{\beta+2}}{\beta+2} + \frac{c.t^{\beta+3}}{\beta+3} \right)\right) \tag{4}$$

Here the higher powers of α is neglected as the value is so small.
Now the solution of equation (2) is

$$I(t)\left(e^{\alpha.t^{\beta}}\right) = -\left(at + \frac{b.t^2}{2} + \frac{c.t^3}{3} + (\alpha)\left(\frac{at^{\beta+1}}{\beta+1} + \frac{b.t^{\beta+2}}{\beta+2} + \frac{c.t^{\beta+3}}{\beta+3}\right)\right) + c_2 \qquad (5)$$

where c_2 is a constant of integration.
Since $I(t_1) = S$ when $t = t_1$ and $I(t_2) = 0$, from equation (5), we have

$$I(t) = \begin{pmatrix} S\left(1 + \alpha.t_1^{\beta} - \alpha.t^{\beta}\right) + a.\left(t_1 - t + \frac{\alpha.t_1^{\beta+1}}{\beta+1} + \frac{\alpha.\beta.t^{\beta+1}}{\beta+1} - \alpha.t_1.t^{\beta}\right) \\ + b.\left(\frac{1}{2}.(t_1^2 - t^2) + \frac{\alpha.t_1^{\beta+2}}{\beta+2} + \frac{\alpha.\beta.t^{\beta+2}}{2(\beta+2)} - \frac{\alpha.t_1^2.t^{\beta}}{2}\right) \\ + c.\left(\frac{1}{3}.(t_1^3 - t^3) + \frac{\alpha.t_1^{\beta+3}}{\beta+3}.+ \frac{\alpha.\beta.t^{\beta+3}}{3(\beta+3)} - \frac{\alpha.t_1^3.t^{\beta}}{3}\right) \end{pmatrix} \qquad (6)$$

Once again the higher powers of α is neglected.
The *total cost* (TC) is given by

$$TC = OC + IHC + DC - SV \qquad (7)$$

where OC-ordering cost, IHC- holding cost, DC-deterioration cost and SV-salvage cost.
Now

1. The Ordering cost = A
2. Inventory holding cost per unit is given by

$$IHC = \int_0^{t_2} h.I(t)dt$$

$$= \int_0^{t_1} h.I(t)dt + \int_{t_1}^{t_2} h.I(t)dt$$

Where

$$\int_0^{t_1}(a_1 + a_2t).I(t)dt = \int_0^{t_1}(a_1 + a_2t).(\gamma - 1)\begin{pmatrix} at + \frac{b.t^2}{2} + \frac{c.t^3}{3} - \\ (\alpha)\left(\frac{a\alpha\beta t^{\beta+1}}{\beta+1} + \frac{b\alpha\beta.t^{\beta+2}}{2(\beta+2)} + \frac{c\alpha\beta.t^{\beta+3}}{3(\beta+3)}\right)\end{pmatrix}dt$$

And

$$\int_{t_1}^{t_2}(a_1 + a_2t).S(t)dt = \int_{t_1}^{t_2}(a_1 + a_2t).\begin{pmatrix} a.\left(t_2 - t_1 + \frac{\alpha.t_2^{\beta+1}}{\beta+1} + \frac{\alpha.\beta.t_1^{\beta+1}}{\beta+1} - \alpha.t_2.t_1^{\beta}\right) \\ + b.\left(\frac{t_2^2 - t_1^2}{2} + \frac{\alpha.t_2^{\beta+2}}{\beta+2} + \frac{\alpha.\beta.t_1^{\beta+2}}{2(\beta+2)} - \frac{\alpha.t_2^2.t_1^{\beta}}{2}\right) \\ + c.\left(\frac{t_2^3 - t_1^3}{3} + \frac{\alpha.t_2^{\beta+3}}{\beta+3}.+ \frac{\alpha.\beta.t_1^{\beta+3}}{3(\beta+3)}\right) - \frac{\alpha.t_2^3.t_1^{\beta}}{3}\end{pmatrix}dt$$

3. The deterioration cost is given by

$$DC = b_1.\left((\gamma - 1)\int_0^{t_1}(a + b.t + c.t^2)dt - \int_{t_1}^{t_2}(a + b.t + c.t^2)dt\right)$$

$$= b_1\left(a.(\gamma.t_1 - t_2) + \frac{b}{2}\left(\gamma.t_1^{\ 2} - t_2^{\ 2}\right) + \frac{c}{3}\left(\gamma.t_1^{\ 3} - t_2^{\ 3}\right)\right)$$

4. Salvage value is given by

$$SV = \gamma_1.CD$$

$$= \gamma_1.b_1\left(a.(\gamma.t_1 - t_2) + \frac{b}{2}\left(\gamma.t_1^{\ 2} - t_2^{\ 2}\right) + \frac{c}{3}\left(\gamma.t_1^{\ 3} - t_2^{\ 3}\right)\right)$$

The Sales Revenue of the system is given by

$$SR = S_r.\left((\gamma - 1)\int_0^{t_1}(a + b.t + c.t^2)dt + \int_{t_1}^{t_2}(a + b.t + c.t^2)dt \right)$$

$$= S_r.\left((\gamma - 1)\left(a.t_1 + \frac{b.t_1^{\ 2}}{2} + \frac{c.t_1^{\ 3}}{3}\right) + \left(a.(t_2 - t_1) + \frac{b.(t_2^{\ 2} - t_1^{\ 2})}{2} + \frac{c.(t_2^{\ 3} - t_1^{\ 3})}{3}\right)\right)$$

Thus the Total Profit of the system is

$$P(t_1, t_2) = \left(\frac{1}{T}\right)(SR - TC)$$

$$= \left(\frac{1}{T}\right)\left(\begin{array}{l} S_r.\left((\gamma - 1)\left(a.t_1 + \frac{b.t_1^{\ 2}}{2} + \frac{c.t_1^{\ 3}}{3}\right) + \left(a.(t_2 - t_1) + \frac{b.(t_2^{\ 2} - t_1^{\ 2})}{2} + \frac{c.(t_2^{\ 3} - t_1^{\ 3})}{3}\right)\right) \\ - \left(\begin{array}{l} A + \int_0^{t_1}(a_1 + a_2 t).I(t)dt + \int_{t_1}^{t_2}(a_1 + a_2 t).S(t)dt \\ + (1 - \gamma_1).b_1\left(a.(\gamma.t_1 - t_2) + \frac{b}{2}\left(\gamma.t_1^{\ 2} - t_2^{\ 2}\right) + \frac{c}{3}\left(\gamma.t_1^{\ 3} - t_2^{\ 3}\right)\right) \end{array} \right) \end{array} \right)$$

$$\therefore P(t_1, t_2) = \left(\frac{1}{T}\right)\left(\begin{array}{l} S_r.\left(\begin{array}{l} (\gamma - 1)\left(a.t_1 + \frac{b.t_1^{\ 2}}{2} + \frac{c.t_1^{\ 3}}{3}\right) \\ + \left(a.(t_2 - t_1) + \frac{b.(t_2^{\ 2} - t_1^{\ 2})}{2} + \frac{c.(t_2^{\ 3} - t_1^{\ 3})}{3}\right) \end{array} \right) \\ - \left(\begin{array}{l} A + \int_0^{t_1}(a_1 + a_2 t).I(t)dt + \int_{t_1}^{t_2}(a_1 + a_2 t).S(t)dt \\ + (1 - \gamma_1).b_1\left(a.(\gamma.t_1 - t_2) + \frac{b}{2}\left(\gamma.t_1^{\ 2} - t_2^{\ 2}\right) + \frac{c}{3}\left(\gamma.t_1^{\ 3} - t_2^{\ 3}\right)\right) \end{array} \right) \end{array} \right) \quad (8)$$

The optimum value of t_1 and t_2 are obtained by solving

$$\frac{\partial}{\partial t_1}P(t_1, t_2) = 0, \ \frac{\partial}{\partial t_2}P(t_1, t_2) = 0 \quad (9)$$

The following conditions are necessary and sufficient to maximize the Total Profit $P(t_1, t_2)$ per unit time

$$\frac{\partial^2}{\partial t_1^2}P(t_1, t_2) < 0 \ , \ \frac{\partial^2}{\partial t_2^2}P(t_1, t_2) < 0 \quad (10)$$

And

$$\left(\frac{\partial^2}{\partial t_1^2}P(t_1, t_2)\right)\left(\frac{\partial^2}{\partial t_2^2}P(t_1, t_2)\right) - \left(\frac{\partial^2}{\partial t_1 \partial t_2}P(t_1, t_2)\right)^2 > 0 \quad (11)$$

Using MATHCAD, the above equations (9) are solved for optimality.

4. Numerical Example

To test the validity of the model, the following values in suitable units, are assumed for various parameters in the model:

$\alpha = 0.01$ $\beta = 1.75$ $\gamma = 5$ $a_1 = 2.5$ $a_2 = 1.0$ $b_1 = 0.5$ $a = 250$ $b = 10$ $c = 0.5$ $\gamma_1 = 3$ $A = 250$ $T = 3$ $t_1 = 0.5$ $t_2 = 1.2$ $S_r = 5$

The optimality conditions given by equations (10) and (11) are satisfied in all types of demand patterns (i.e., accelerated growth, retarded growth, accelerated decline and retarded decline in demand models). Table-1 shows the results of various models. It is observed that the behavior of these models is similar.

Table 1

Model	t_1	t_2	$P(t_1, t_2)$
Accelerated Growth Model	1.591	2.089	900.695
Retarded Growth Model	1.574	2.063	900.316
Accelerated Decline Model	1.431	1.853	888.532
Retarded Decline Model	1.443	1.870	888.911

5. Sensitivity Analysis

We now study sensitivity of the models developed to examine the implications of underestimating and overestimating the parameters individually and all together on optimal value of total profit. The Sensitive analysis is performed by changing each of the parameter by -50%, -20%, +20% and +50% taking one parameter at a time and keeping the remaining parameters are unchanged and finally all parameters are considered. Since all models show similar results, we will present only the sensitivity for accelerated growth model. The results are shown in *Table*-2. The following observations are made from this table:

1. The profit function $P(t_1, t_2)$ of the system increases (decreases) with an increase (decrease) in the values of the parameters γ, a, b, c, b_1, γ_1 and Sr while it decreases (increases) with increase (decrease) in the values of the parameters α, β, a_1 and a_2.
2. However, the profit $P(t_1, t_2)$ is highly sensitive to the changes in the values of the parameters γ, a, and S_r, moderately sensitive to the changes in a_1, a_2, b_1 and γ_1 and slightly sensitive to the changes in the values of the parameters α, β, b and c.
3. As expected, the increase (decrease) in the variable holding cost decreases (increases) in the value of the profit function $P(t_1, t_2)$ of the system.
4. Similarly the increase (decrease) in the salvage value increases (decrease) the profit of the system.
5. The profit function $P(t_1, t_2)$ of the system is highly sensitive to the changes in the values of all parameters taken together in the model.

The variations of $P(t_1, t_2)$ with respect to the values of some important parameters is shown in Figure 1 the variations of $P(t_1, t_2)$ with respect to t_1 and t_2 are shown in Figure-2 and Figure.-3 respectively.

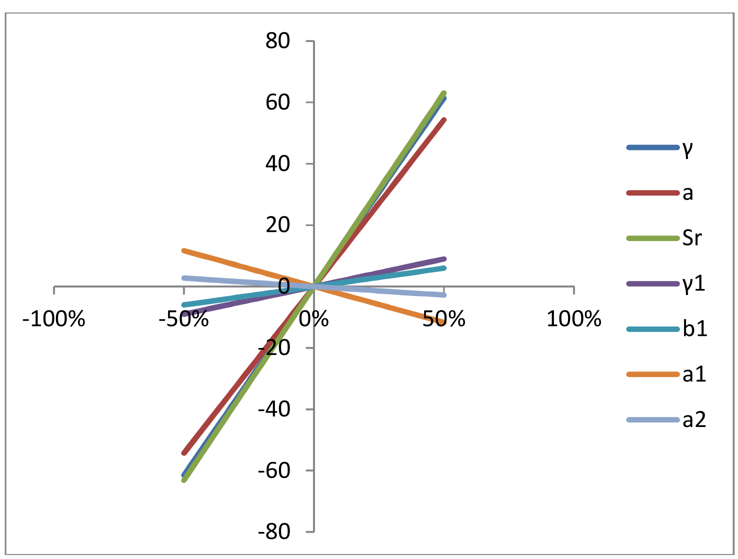

Figure 1

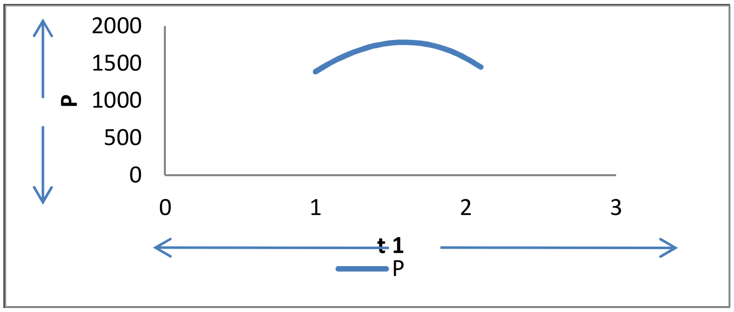

Figure 2

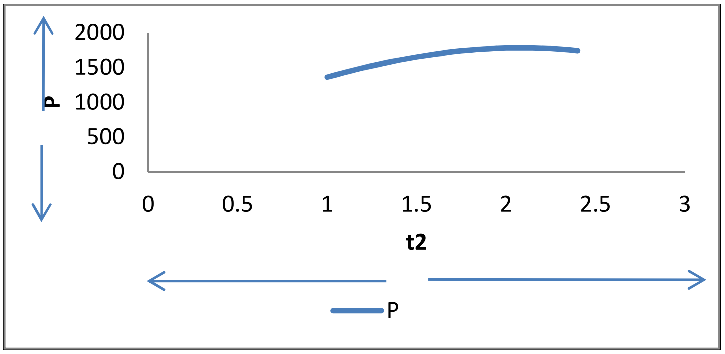

Figure 3

Table 2 *If a>0, b>0, c>0*

parameter	% Change in Parameter	t_1	t_2	P	change%		
					$\%t_1$	$\%t_2$	%P
α	50	1.601	2.095	900.412	0.6285	0.2872	-0.0314
	20	1.595	2.091	900.582	0.2514	0.0957	-0.0125
	-20	1.588	2.086	900.808	-0.1886	-0.1436	0.0125
	-50	1.582	2.082	900.978	-0.5657	-0.3351	0.0314
β	50	1.605	2.094	900.512	0.8799	0.2393	-0.0203
	20	1.596	2.091	900.608	0.3143	0.0957	-0.0097
	-20	1.587	2.086	900.805	-0.2514	-0.1436	0.0122
	-50	1.582	2.083	901.019	-0.5657	-0.2872	0.0360
γ	50	1.59	2.087	1454	-0.0629	-0.0957	61.4309
	20	1.591	2.088	1122	0.0000	-0.0479	24.5705
	-20	1.592	2.09	679.249	0.0629	0.0479	-24.5861
	-50	1.597	2.094	347.081	0.3771	0.2393	-61.4652
a	50	1.561	2.043	1390	-1.8856	-2.2020	54.3253
	20	1.576	2.066	1096	-0.9428	-1.1010	21.6838
	-20	1.616	2.124	705.106	1.5713	1.6754	-21.7153
	-50	1.696	2.243	411.722	6.5996	7.3719	-54.2884
b	50	1.636	2.154	903.641	2.8284	3.1115	0.3271
	20	1.609	2.114	901.873	1.1314	1.1967	0.1308
	-20	1.574	2.064	899.517	-1.0685	-1.1967	-0.1308
	-50	1.55	2.028	897.749	-2.5770	-2.9201	-0.3271
c	50	1.596	2.095	900.79	0.3143	0.2872	0.0105
	20	1.593	2.091	900.733	0.1257	0.0957	0.0042
	-20	1.59	2.086	900.657	-0.0629	-0.1436	-0.0042

	-50	1.587	2.082	900.6	-0.2514	-0.3351	-0.0105
a_1	50	1.257	1.654	795.201	-20.9931	-20.8234	-11.7125
	20	1.442	1.895	858.498	-9.3652	-9.2867	-4.6849
	-20	1.766	2.315	942.893	10.9994	10.8186	4.6850
	-50	2.083	2.727	1006	30.9239	30.5409	11.6915
a_2	50	1.386	1.81	875.648	-12.8850	-13.3557	-2.7809
	20	1.497	1.96	890.676	-5.9082	-6.1752	-1.1124
	-20	1.711	2.253	910.714	7.5424	7.8506	1.1124
	-50	1.976	2.622	925.742	24.1986	25.5146	2.7809
b_1	50	1.697	2.136	954.673	6.6625	2.2499	5.9929
	20	1.634	2.108	922.286	2.7027	0.9095	2.3971
	-20	1.548	2.069	879.104	-2.7027	-0.9574	-2.3971
	-50	1.482	2.04	846.717	-6.8510	-2.3456	-5.9929
γ_1	50	1.749	2.159	981.662	9.9309	3.3509	8.9894
	20	1.655	2.117	933.082	4.0226	1.3404	3.5958
	-20	1.526	2.06	868.308	-4.0855	-1.3882	-3.5958
	-50	1.427	2.015	819.729	-10.3080	-3.5424	-8.9893
S_r	50	2.16	2.936	1469	35.7637	40.5457	63.0963
	20	1.815	2.423	1128	14.0792	15.9885	25.2366
	-20	1.367	1.753	673.264	-14.0792	-16.0843	-25.2506
	-50	1.022	1.229	332.118	-35.7637	-41.1680	-63.1265
All	50	1.796	2.195	3735	12.8850	5.0742	314.6798
	20	1.665	2.124	1717	4.6512	1.6754	90.6306
	-20	1.521	2.057	392.202	-4.3997	-1.5318	-56.4556
	-50	1.417	2.012	23.872	-10.9365	-3.6860	-97.3496

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