An Inventory Model with Queuing theory for Improving Supply Chain Strategy through Substitution Flexibility and Simultaneous Ordering

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This study presents an inventory model that leverages substitution flexibility, a strategy often beneficial in multi-product inventory systems. Specifically, we focus on a scenario featuring two substitute products (i.e. smartphones and tablets) and negligible lead times. We introduce stochastic parameters to represent customer demands for both products. Also, we incorporate queuing theory, a powerful analytical tool for optimizing inventory systems. Our goal is to minimize the combined costs associated with ordering, holding inventory, and the simultaneous ordering process. To achieve this optimisation, we employ the ABC-GOA (Artificial Bee Colony-Gravitational Optimization Algorithm) technique, a state-of-the-art optimisation approach. This methodology is applied to streamline the supply chain, reducing ordering costs, holding costs, and enhancing efficiency in the simultaneous ordering process. The result is verified by real time examples.

Keywords: Inventory Model, Substitution Flexibility, Simultaneous Ordering, Queuing Theory, Supply Chain Strategy, ABC-GOA.

1. Introduction

The introduction emphasizes that effective inventory management is crucial for success in supply chain management. It highlights that inventory is not just a cost but a strategic asset that helps meet customer demands quickly. The paper discusses the need to balance the costs associated with maintaining inventory against the benefits it provides. This balance is essential for achieving efficient inventory management. The concept of flexibility in inventory management is introduced as a key strategy. This flexibility can be achieved through various methods, such as using common components, lateral transhipments, and simultaneous ordering. A significant aspect of flexibility is risk pooling, which allows for the consolidation of demand for multiple products onto a flexible inventory. This approach can reduce the need for safety stock and lower inventory holding costs, although it also presents challenges and trade-offs. Overall, the introduction sets the stage for discussing an innovative inventory model that incorporates queuing theory and substitution flexibility to improve supply chain strategies. Effective inventory management is all about striking the right balance between the costs incurred and the benefits reaped from maintaining a stockpile of goods. One strategy to achieve this equilibrium is by introducing flexibility into the inventory system (Ahiska & Kurtul, 2014) (Alimardani et al., 2013). Flexibility in inventory management is a multifaceted concept that can be realized through various means, including the use of common components, lateral transhipments, and postponement, simultaneous ordering. The cornerstone of flexibility lies in the concept of risk-pooling, which allows the demand for multiple end products to be consolidated onto a flexible inventory (Arda & Hennet, 2006). This approach, while reducing the need for safety stock and inventory holding costs, comes with its own set of challenges and trade-offs. In this context, it's essential to understand that flexibility often comes at a cost, which encompasses both a product cost premium and additional adjustment costs. This realization has spurred research in the domain of substitution systems, where flexible inventory is employed only when the regular, more cost-effective items are depleted. This approach is often referred to as "tailored flexibility" or "tail-pooling" (Aria Nezhad et al. 2013).

1.1 The Role of Flexibility in Inventory Management

Flexibility in inventory management refers to the capability of adapting to changes in demand, supply, or other factors efficiently. It can be achieved through various means, including (Bahri & Tarokh, 2012):

- **Common Components:** Using interchangeable parts or components in the production process to reduce the variety of items in inventory.
- **Lateral Transhipments:** The transfer of inventory between different locations or nodes in a supply chain to fulfil demand when one location experiences a shortage.
- **Postponement:** Delaying the final configuration of products until customer demand is known, reducing the need for storing multiple finished products(Bayındır, 2003).
- **Simultaneous Ordering:** Placing orders for multiple products at the same time to optimize supply chain operations.

This research paper focuses on the integration of substitution flexibility, simultaneous ordering, and queuing theory within the context of supply chain management. We will explore the various settings in which substitution flexibility can be applied, the benefits it offers in terms of cost reduction and improved customer service, and the challenges and trade-offs associated with its implementation (Deflem & Van Nieuwenhuyse, 2012). Furthermore, we will delve into the role of queuing theory as a tool to enhance the efficiency and decision-making in supply chain operations. In summary, this paper aims to shed light on the strategic potential of substitution flexibility, simultaneous ordering, and queuing theory integration in inventory management. By understanding the concept of tailored flexibility and exploring its application in diverse scenarios, supply chain professionals can develop more agile and cost-effective inventory management strategies, ultimately improving their overall supply chain performance (Herer et al., 2004).

2. Related Work

Hill et al. (2007) Gives an economic order quantity model with planned shortages. This paper discusses how to handle planned shortages within the EOQ model and offers insights into managing inventory in the presence of substitution flexibility. Also, by Hosseini et al. (2013) delves into various inventory control techniques, including order policies that incorporate queuing theory, providing valuable insights into supply chain strategies. While Sapna Isotupa (2006) focused on inventory deterioration, this paper highlights the importance of modelling different aspects of inventory, which can be integrated into a comprehensive model. Also (Krommyda et al., 2015) provided a practical application of simultaneous ordering and substitution flexibility within a supply chain context. Further (Liu & Lee, 2007) , it gives a model in which Inventory management under substitution flexibility and price-dependent demand. This research explores inventory models that incorporate substitution flexibility and demand that depends on the price, emphasizing the relevance of these concepts in supply chain strategies. By (Nagarajan & Rajagopalan, 2008) suggesting a comprehensive inventory model with queuing theory integration for improving supply chain strategy through substitution flexibility and simultaneous ordering. A closely related research stream studies customer-driven substitution in the context of assortment planning.

Gioia & Minner (2023) highlighted the benefits of inventory pooling in multi-channel, multi-echelon systems under stochastic demand, emphasizing the impact of FIFO/LIFO policies and base stock heuristics. They conclude that perishable goods favour constant orders, while demand correlations and imbalances require careful optimization. Also, Alnowibet (2022) introduced a stochastic inventory model that incorporates long lead times, impatient customers, and random replenishment quantities, analysed using Markov chains. The study optimizes key metrics like average inventory, lost sales, and customer waiting times, emphasizing the balance between supply and demand. The model's accuracy is validated, with the potential for extension to more complex queuing-inventory scenarios.

2.1 Review on Queuing theory in Inventory

Pandey & Sharma (2024) explored a production supply chain inventory model that integrates queuing theory, focusing on carbon emissions and the learning effect. It aims to minimize total inventory costs while addressing stochastic demand and the impact of a carbon emissions tax. The authors validate their model through numerical examples and conduct a sensitivity analysis to assess the effects of various inventory parameters in industrial applications. Further Pandey, J et al. (2024) focused on supply chain management, particularly in the context of fluctuating demand and carbon emissions. This paper highlights that stable demand is crucial for the smooth functioning of supply chain operations. The paper focuses on integrating environmental considerations into supply chain management through advanced modelling techniques, emphasizing the importance of addressing both demand variability and carbon emissions. Also, Rashid et al. (2015) presented a mathematical model for an inventory control system that incorporates stochastic parameters for customer demands and supplier service times, solved using queuing theory for single-item scenarios. The model is then extended to multi-item inventory systems, and a heuristic algorithm is developed to address the complexity of the problem.

Marsudi & Shafeek (2014) conducted a queuing analysis to evaluate the performance of a multi-stage production line, specifically for a company producing battery covers for cameras. The study aims to enhance resource planning and improve efficiency by utilizing real-life data from the production process. Key performance measures such as utilization, idle workstation percentage, and expected queue times are analyzed to provide actionable insights for managers.

2.2 Review on Inventory Systems

In a perpetual inventory system, the inventory levels of the items are continuously monitored. When the level reaches or falls below a predetermined reorder point, a replenishment order is triggered. The literature has thoroughly analysed both (S-1, S) and (R, Q) policies. This overview focuses on studies that assume total pooling, which means that none of the inventory is specifically held for devoted demand (1).

Table 1 *Review on Inventory System*

Reference	Kind of Flexibility	Description of Model	Objective
(Olsson, 2010)	Product Substitution	The model used is the (S-1, S) model, which considers exponential replenishment lead time and involves two goods. It also takes into account one-way substitution and lost sales.	Optimise the difference between sales income and the combined costs of purchasing and inventory holding.
(Parlar, 1997)	Product Substitution	Two products, one-way substitution, exponential lead time for replenishing Backlog Substitution based on order delivery and demand arrival, (S-1,S) model	Examine the impact on average backorder, fill rate, average inventory, and average demand substitution likelihood.
(Tan & Karabati, 2013)	Transhipment	Exponential lead time for restocking n-location unidirectional lateral transhipment Backlogs and lost sales are both permitted under the S-1, S, and R, Q policies.	Reduce the total amount spent on holding, transhipment, and backorder (or lost sales) expenses.
(Yu & Dong, 2014)	Product commonality	Discounted costs: No product commonality versus pure product commonality - Preorder	Reduce the total cost of acquisition, holding, and shortfall.
(Ye, 2014)	Product commonality	Product redundancy as a fallback - Cost savings - Missed sales	Reduce the total cost of acquisition, holding, and shortfall.
(Tokta-Palut & Ülengin, 2011)	Component commonality	Discounted costs: No component commonality versus pure component commonality	Reduce the total cost of acquisition, holding, and shortfall.
(Tlili et al., 2012)	Component commonality	Substitution of components	Optimise total sales income less purchase costs, as well as salvage value.
(Tibben-Lembke & Bassok, 2005)	Flexible resource	Ascertain the resources' capacity level.	maximise revenue from operations less capacity cost
(Teimoury et al., 2010)	Flexible resource	Identify the resources' capacity level and the best raw material ordering strategy.	Optimise income less the total of holding, purchasing, investing, and shortfall costs.

2.3 Research Gap

The paper's inventory model relies on certain assumptions, such as negligible lead times and the availability of substitute products. These assumptions may not hold true in real-world scenarios, potentially limiting the model's applicability in more complex supply chain environments. In summary, while the paper presents a novel approach to inventory management, its limitations include assumptions that may not hold in practice, a narrow focus on two products, and a lack of empirical validation, which could affect the model's applicability and effectiveness in real-world scenarios.

2.4 Motivation

The motivation behind this research stems from the increasing complexity and competitiveness of today's business environment, particularly in supply chain management. This research highlights the growing intricacy of tactical supply chain planning. As businesses face more competition, the need for effective strategies to manage supply chains becomes critical.

3. Model Development and Analysis

Assumptions

The paper presents an inventory model that incorporates various assumptions to optimize supply chain strategies through substitution flexibility and simultaneous ordering.

- There are two substitute products (smartphones and tablets), which allow for flexibility in inventory management. And substation cost according to standard norms are considered with the help of renowned citation Seyedhoseini et al. (2015).
- Lead times for ordering the products are negligible.
- This model incorporates stochastic parameters to represent customer demands for both products (smartphones and tablets).
- Minimizing combined costs associated with ordering, holding inventory, and the simultaneous ordering process.
- Employ the Poisson distribution to describe the demand for each product and employ a Markov process.

Model Description

This study delves into a system of inventory characterized by two replacement items, emphasizing a strict no-shortage policy, and accounting for replacement costs associated with both products. The states within the system are represented as (i, j), with i and j symbolizing the inventory levels of each product type. The transition diagram, as depicted in Figure 1, categorizes states into three sets: Set A and Set C The amounts of inventories for every kind of goods encompass states with singular inventory types, while States having both Inventories are represented by Set B, product types, without any substitution.

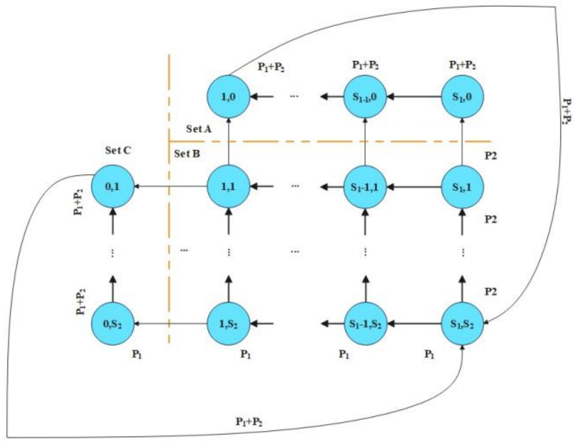


Figure 1 *Transition Diagram*

To construct a comprehensive model for this inventory system, four lemmas have been introduced to calculate the likelihood of the stable state relative to the state's probability (S_1, S_2). In the following discussion, we will use the notation $\pi_{i,j}$ to represent the steady-state State (i, j) probability. We also indicate the rate of demand for the initial category of products and the rate of demand for the second.

Lemma 1 deals with scenarios where flows arrive through a single state, move only right or top, and reach a node (*i*, *j*). The arrival flows at *(i*, *j*) are the sum of flows from the node (S_1, S_2) via multiple directions. $IF_{i,j}$ represents the interval flow for (*i*, *j*), $IF_{i,j}^k$ is the arrival flow via path k, and N is the total number of paths connecting (S_1, S_2) to (*i*, *j*). This lemma is foundational in understanding flow dynamics within the network.

$$IF_{i,j} = \sum_{k=1}^{N} IF_{i,j}^k \quad (1)$$

Proof: Since the arrival flow in this network exclusively passes through node (S_1, S_2), it logically follows that every flow directed towards the state (i, j) originates from the state (S_1, S_2). Given the constraint it only moves upward or to the right side, with every path joining a node (S_1, S_2) to node (i, j), reflecting a unique flow and is characterised by *i*-1 lateral moves and *j*-1 vertical moves. Moreover, these flows only go along certain pathways.

Therefore, the interval flow at state (*i*, *j*), denoted as $IF_{I,J}$ can be determined by summing the flows traversing these specific paths. This calculation can be expressed using Equation 1.

Lemma 2, In a queuing network with a lateral output rate with value, $IF_{I,J}$ and a vertical with P, the $IF_{I,J}$ can be calculated by following equation

$$IF_{i,j} = \left(\left(\frac{P_1{}^{s_1-i}P_2{}^{s_2-j}}{(P_1+P_2)^{s_1+s_2-i-j}}\right)\binom{s_1+s_2-i-j}{s_2-j}\right)IF_{S_1S_2} \tag{2}$$

Proof: For $IF_{I,J}$, the probable vertical move is $\frac{P_2}{P_1+P_2}$, and the probable lateral move is $\frac{P_1}{P_1+P_2}$. If flow moves vertically it will be $\frac{P_2}{P_1+P_2}$, times of coefficient, and if it moves laterally, then $\frac{P_1}{P_1+P_2}$. Also, between node (S_1, S_2) and node $(i,\ j)$, there are $s_2 - j$ vertical $s_1 - j$ lateral moves. The input flows for each path in (i, j) is

$$\left(\frac{P^{i-1}\mu^{i-1}}{(\mu+P)^{j+i-2}}\right)IF_{1,1} \tag{3}$$

And the total number of paths between (s_1, s_2) and (i, j) is:

$$\binom{s_1+s_2-i-j}{s_2-j} \tag{4}$$

The formula below may be derived by using Lemma 1, equations 3 and 4.

$$IF_{i,j} = \sum_{k=1}^{N} IF_{i,j}^{k} = \left(\frac{P_1{}^{s_1-i}P_2{}^{s_2-j}}{(P_1+P_2)^{s_1+s_2-i-j}}\right)$$

$$IF_{s_1s_2}\sum_{k=1}^{N} 1 = \left(\left(\frac{P_1{}^{s_1-i}P_2{}^{s_2-j}}{(P_1+P_2)^{s_1+s_2-i-j}}\right)\binom{s_1+s_2-i-j}{s_2-j}\right)IF_{S_1S_2} \tag{5}$$

For this Markov process, the output rate is $P_1 + P_2$ and for steady-state probabilities, the following equation becomes true:

$$IF_{i,j} = (P_1 + P_2)\pi_{i,j} \tag{6}$$

Where $\pi_{i,j}$ is the steady-state probabilities of state (i, j).
So $IF_{s_1s_2} = \frac{IF_{s_1s_2}}{(P_1+P_2)}$ and for the set C, we can find a steady state using equation (7) given below

$$\pi_{i,j} = \left(\left(\frac{P_1{}^{s_1-i}P_2{}^{s_2-j}}{(P_1+P_2)^{s_1+s_2-i-j}}\right)\binom{s_1+s_2-i-j}{s_2-j}\right)\pi_{S_1S_2} \tag{7}$$

Now, lemma 3 is proposed to find the steady state probability of set A.

Lemma 3: For set A, the steady state probabilities can be found by –

$$\pi_{i,0} = \sum_{r=0}^{s_1-i} \pi_{s_1-r,1}\ \frac{P_1}{P_1+P_2} =$$

$$= \binom{s_2+s_1-i-j}{s_2-j} \pi_{s_1,s_2}. \frac{P_1{}^{s_1-r} P_2{}^{s_2-1}}{(P_1+P_2)^{s_1+s_2-r-1}} \frac{P_1}{P_1+P_2} \tag{8}$$

Proof: All the Flows arrive in set A, come from j = 1 state, and with $P_1 + P_2$ lateral rate only. Hence for the $(i, 0)$ state following equation is true.

$$\mu(\pi_{i,0}) = D(\pi_{i,1} + \pi_{i+1,1} + \cdots + \pi_{s_{1,1}}) \tag{9}$$

Lemma is proved by considering lemma 2 and equation (9).
Similarly, we can find the steady state probabilities of set C as follows

$$\pi_{0,j} = \sum_{r=0}^{s_2-j} \pi_{0,s_2-r} \frac{P_1}{P_1 + P_2}$$

$$= \sum_{r=0}^{s_1-i} \binom{s_1+s_2-r-1}{s_2-r} \pi_{s_1,s_2}. \frac{P_1{}^{s_1-1} P_2{}^{s_2-r}}{(P_1+P_2)^{s_1+s_2-r-1}} \frac{P_2}{P_1+P_2} \tag{10}$$

Lemma 4 For product 1 and product 2, the amount of substitution is given by:

$$\sum_{j=1}^{s_2} \pi_{1,j}.j.\left(\frac{P_1}{P_1+P_2}\right)^2 \tag{11}$$

$$\sum_{i=1}^{s_1} \pi_{i,1}.i.\left(\frac{P_2}{P_1+P_2}\right)^2 \tag{12}$$

Proof: state is changed from $\pi_{1,j}$ to $\pi_{0,j}$ by probability $\frac{P_1}{P_1 + P_2}$ and then, for the demand of product 1 $\frac{P_1.j}{P_1 + P_2}$ substitution inventory to be used. Hence substitution amount for product 1 and product 2 are calculated by equations (11) and (12), respectively.

3.1 Objective Function

When dealing with inventory systems that involve substitution flexibility and simultaneous ordering, we can formulate an objective function along with demand constraint, order constraint for individual product, substitution constraint, budget constraint, capacity constraint. The objective function aims to optimize a cost metric, such as the total cost, while the constraints ensure that the system operates within certain limitations.

$$Z = \sum_{i=1}^{n}\left[(C_i * Q_i) + (h_i * \sum_{j=1}^{n} p_{i,j})\right] + A * (D_{total}) + \sum_{i=1}^{n}[C_r * Q_i] \tag{13}$$

Where,

n	**Number of products**
c_i	Price per product unit i.
Q_i	Order quantity for product i.
H_i	Keeping the product's cost per unit i.
$P_{i,j}$	Steady-state probability of being in state (i, j) for product i.
A	Simultaneous ordering cost.
D_{total}	Total demand for all products.
C_r	Replacement cost per unit for each product

Demand Constraint: Ensure that the total ordered quantity matches the total demand for all products involved in the simultaneous order.

$$Q_1 + Q_2 \dots \dots + Q_n = D_{total}$$

Ordering Constraints for Individual Products: Constraints to ensure that order quantities for individual products meet their respective demands.

$$\begin{gathered} Q_1 = D_1 \\ Q_2 = D_2 \\ \vdots \\ Q_n = D_n \end{gathered}$$

Substitution Constraints: If there are substitution options, we can define constraints to limit the substitution ratio between products. For example, if product 2 can substitute for product 1.

$$Q_2 \leq S * Q_1$$

Budget Constraints: If there is a budget constraint, we can restrict the total cost of the order

$$c_1 * Q_1 + c_2 * Q_2 + \cdots + c_n * Q_n < Budget$$

Capacity Constraints: If there are limitations on storage or transportation capacity, we can set constraints to ensure that the total ordered quantity doesn't exceed these limits.

$$Q_1 + Q_2 + \cdots + Q_n \leq Capacity$$

Hybrid Optimization Algorithm

The hybrid optimisation method is a novel optimisation strategy that is presented in this study. This method is a combination of the Artificial Bee Colony (ABC) and the Grasshopper Optimisation method (GOA). This is a fictitious rendition of the recommended algorithm

Algorithm 2: Algorithm for hybrid optimisation

Step 1: A population should be chosen that falls between the given minimum and maximum values from (1)

Step 2: alterations in worker bees' positions using equation (2)

Step 3: Analyse the health and vitality of the bees using equation (3)

Step 4: During the observer bee phase, update positions using a greedy strategy and equation (4)

Step 5: In the observer bee stage, update positions using an avaricious approach and

- Each bee should be assessed for its fitness,
- its status should be updated by equation (5)

Step 6: Go to step 2 if the conversion is unsuccessful.

The hybrid technique's efficacy in handling challenging optimisation problems is shown by comparing its performance to that of the individual ABC and GOA algorithms as well as other cutting-edge optimisation methods. Utilize ABC-GOA, a powerful inventory optimization technique, to minimize the total cost, which comprises ordering costs, holding costs, replacement costs, and simultaneous ordering costs. By customizing the objective function with appropriate parameter weights, you can address your organization's unique priorities and constraints, including demand, budget, capacity, and substitution constraints. This approach optimizes inventory management, enhancing cost-effectiveness and overall operational efficiency, making it ideal for businesses seeking tailored solutions to their inventory challenges.

4. Results

To assess the efficacy of implementing substitution along with independent ordering for two products, a comprehensive evaluation of cost-efficiency is conducted.

Table 2 *Example*

C_2	C_1	D_2	D_1	D_0	A	A_2	A_1	h_2	h_1
25	25	15	7	250	120	40	40	40	12

Table 3 *Comparison between using Substitution and Independent Ordering*

		Product1	Product2
Substitution	Order quantity	12	30
	Cosy of inventory system	410	600
	Substitution costs	36	0
Independently ordering	Order quantity	20	40
	Cosy of inventory system	554	800
	Substitution costs	0	0

Table 3 compares the effectiveness of substituting when placing separate orders for Product 1 and Product 2. In the substitution scenario, order quantities of 12 for Product 1 and 30 for Product 2 lead to a cost-effective inventory system with a total cost of $410, including a minimal substitution cost of $36. Conversely, in the independently ordering scenario, order quantities of 20 for Product 1 and 40 for

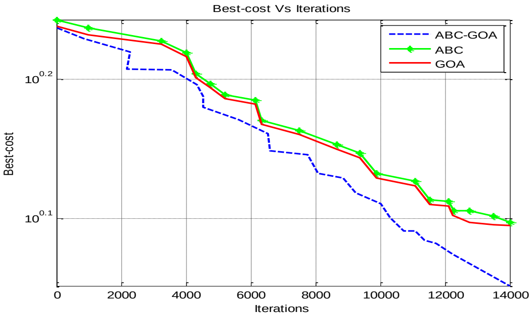
Product 2 result in a higher total cost of $554, with no substitution costs. The Table highlights the cost-saving advantage of utilizing substitution for these products.

Real Time Example

This Table provides a side-by-side comparison of the two inventory management scenarios for smartphones and tablets, showing parameter values, substitution costs, simultaneous ordering costs, and the total cost of the inventory system for each scenario. In this example, Independently Ordering results in a significantly lower total cost due to the absence of additional substitution costs.

Scenario	Substitution	Independently Ordering
Smartphones (Product 1)		
Cost per Unit ($)	$400	$400
Monthly Demand (units)	50 units	50 units
Holding Cost per Unit ($/month)	$10	$10
Replacement Cost ($)	$420	$420
Tablets (Product 2)		
Cost per Unit ($)	$600	$600
Monthly Demand (units)	40 units	40 units
Holding Cost per Unit ($/month)	$15	$15
Replacement Cost ($)	$650	$650
Substitution Cost	$44,000	0 (No Substitution)
Simultaneous Ordering Cost	$30 + $40 = $70	$70
Total Cost of Inventory System	$86,030	$44,070

In this study, we conducted a comparative evaluation of three optimization algorithms: ABC-GOA, ABC, GOA. Over the course of iterations, we analyzed how each algorithm optimized total cost, which comprises ordering costs, holding costs, and simultaneous ordering costs. This evaluation aimed to identify the most efficient algorithm for achieving efficient and cost-effective management of inventory models.



The presented Figure depicts the iteration-wise best cost values for three optimization techniques: ABC-GOA, ABC, and GOA. Notably, our proposed ABC-GOA consistently outperformed the others by achieving the minimum best cost. In the realm of cost optimization, "minimum" is a paramount goal, indicating potential for heightened cost-effectiveness and profitability. This underscores the effectiveness of the ABC-GOA approach in enhancing optimization outcomes, making it an advantageous choice for applications where minimizing costs is a critical objective, such as in supply chain management.

5. Conclusion

In summary, this study highlights the escalating intricacy of tactical supply chain planning in the intensely competitive contemporary business landscape, emphasizing the need for innovative approaches to enhance profitability. It introduces a robust inventory model that capitalizes on substitution flexibility, particularly advantageous in multi-product inventory systems, with a focus on scenarios involving two substitute products and minimal lead times. The study explores the critical aspect of simultaneous ordering in supply chain optimization. By incorporating stochastic parameters to represent customer demands and leveraging queuing theory, it constructs a comprehensive mathematical model aimed at minimizing combined costs encompassing ordering, inventory holding, and simultaneous ordering. The application of the cutting-edge ABC-GOA optimization technique significantly streamlines the supply chain, resulting in cost reduction, improved efficiency, and heightened competitiveness. Ultimately, this paper is significant, offering strategies for cost reduction, improved customer service, and enhanced decision-making in supply chain management, while also providing a foundation for future research in the field.

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