A Note on, “How to Share the Gains of Collaboration?”

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We present some insights on collaborative gain sharing based on “Going to the Beach Exercise” from a recent paper. We propose two gain sharing algorithms in the case of actual carpooling with the car’s wear and tear taken into consideration. We provide algebraic expressions for the volume-based allocation rule, the Shapley value and also for our proposed rules with mileage or wear and tear factored in. Furthermore, we show that our two proposed rules are equivalent for a special case. We also discuss the scenario of how frequent travelers may negotiate gain sharing to their advantage if they have privileges such as free parking at the beach.

Keywords: Gain Sharing, Carpooling, Gamification, Supply Chain, Algebraic Expressions, Collaboration

1. Introduction

The business model built on the principle of 'Gains of Collaboration' has potential for substantial financial benefits, and increased productivity. Applications embracing this concept encompass various domains, such as carpooling in transportation (Liu et al, 2020), shared warehousing in logistics (Ding & Kaminsky, 2019), and inter-organizational collaboration in strategic alliances (Gulati et al, 2012). Examining case studies and simulations based on datasets in carpooling offer theoretical insights into the intricate behaviors of collaborative systems (Wang et al, 2017).

The “Going to the Beach Exercise” was described in a recent article (Barbarino & Boute, 2023). The motivation for this exercise is that different shippers bundle their orders and procure transport services from the same carrier. This enables shippers to negotiate better with full truckload (FTL) service instead of individually with less-than-truckload (LTL) service (Guajardo & Ronnqvist, 2016). This is similar to carpooling with environmental benefits such as reducing carbon footprints and emissions (Caulfield, 2009). An additional benefit of carpooling is the high-occupancy vehicle (HOV) lanes which are typically faster than the often-congested general-purpose lanes (Boysen et al, 2021). The difference is that carpooling by itself does not involve a third-party carrier as in the case of transportation. One of the shippers becomes a carrier and may be compensated for wear and tear of the transport vehicle being used.

We implemented the "Going to the Beach Exercise" (Barbarino & Boute, 2023) in an MBA class for two consecutive semesters. This exercise is provided in the Appendix. We explained the traditional gain sharing methods called the *volume-based rule* and *Shapley value* but asked our class to use their own methods to arrive at mutually agreeable solutions. During this exercise, one of the MBA teams treated this as an actual carpooling exercise and argued for lower costs for the car that is driven to the beach. After the class discussion, we analyzed it further and developed two gain sharing algorithms for this actual carpooling scenario. We propose these rules and present algebraic expressions. We also prove that these two rules are equivalent for a special case. In this paper, we also consider the scenario of the frequent traveler with free parking privileges and present a gain sharing rule for this scenario too.

The two semesters of MBA had small class sizes with the following mix of students.

Table 1 *Class size and other Information*

Period of Study	Class Size	Male	Female	Full-time MBA students	Working Professionals
Fall 2024	10	5	5	8	2
Spring 2025	9	5	4	7	2

The carpooling exercise with wear and tear is a special case of the "Going to the Beach Exercise" and complements this exercise. Since our proposed gain sharing rules are simple, this exercise can be used in any quantitative class that discusses supply chain management or logistics.

2. Gain Sharing Rules and Algebraic Expressions

The volume-based rule and Shapley value method were explained for the "Going to the Beach Exercise" (Barbarino & Boute, 2023). For the benefit of the reader, the exercise is presented in the appendix and the rules explained again here. Consider a scenario with 3 cars. One car is yellow (2) and the other two cars are white (1). The numbers in parenthesis indicate the number of people in these cars. It costs €100 per car to go to the beach and this cost is shared by the number of people it carries. This means it will cost €100 for each of the two white cars and €50 per person for the yellow car. As per this exercise, each car has a seating capacity of five people. To reduce costs, these people may choose to carpool and go in one car.

Volume-Based Allocation Rule: Cost/person = €100/n where n is the number of people sharing a car. For the above scenario of four people, it costs €25/person. This is a simple rule but not perceived to be fair because it does not take prior efficiencies into account. By prior efficiencies, we mean that the cost for people in the yellow car was €50/person and the cost for the white car was €100/person prior to this collaboration. So, the volume-based allocation rule simply does not take this into consideration. Clearly, the people in the white cars benefit more (a 75% reduction in costs) with this rule upon collaboration. As for the yellow car, the cost reduction is only 50% per person.

Shapley Value: A single car costs €100 as opposed to €300 if all three cars were to go to the beach i.e., a saving of €200. The Shapley value distributes savings equally amongst all cars. Thus, the cost of each car decreases by €66.6, and it costs (100-66.6)/n per person where "n" is the original number of people in each car. Thus, for the yellow car, it will cost (100-66.6)/2 = €16.6 per person. For white cars, it will be (100-66.6)/1 = €33.3 per person. This is an exact decrease of 66.6% per person for all cars. Since Shapley value rewards each person with the same relative benefit, it is perceived to be a fair rule for collaborative gain sharing. We present the costs of both rules in Table 2 (like the Table presented by Barbarino and Boute, 2023).

Table 2 *Collaboration of One Yellow Car and Two White Cars*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Volume-based allocation	Cost (€) pp after collaboration: Shapley value
Yellow (2)	50	25 (– 50%)	16.6 (– 66.6%)
White (1)	100	25 (– 75%)	33.3 (– 66.6%)
White (1)	100	25 (– 75%)	33.3 (– 66.6%)

We now present three types of collaborations and provide algebraic expressions for each in Tables 3, 4, and 5. Here, we do not associate a color to a car and no longer assume the seating capacity of a car as five.

Collaboration 1: All cars have the same number of people (identical cars).

Table 3 *Collaboration of Three Identical Cars with "n" People in each car*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Volume-based allocation	Cost (€) pp after collaboration: Shapley value
Car 1 (*n*)	100/*n*	100/3*n* (– 66.6%)	(100-66.6)/*n* (– 66.6%)
Car 2 (*n*)	100/*n*	100/3*n* (– 66.6%)	(100 - 66.6)/*n* (– 66.6%)
Car 3 (*n*)	100/*n*	100/3*n* (– 66.6%)	(100 – 66.6)/*n* (– 66.6%)

Note that both Shapley value and volume-based allocations result in identical costs for everyone. This holds true for the collaboration of any number of cars as long as each car has the same number of people.

Collaboration 2: Collaboration of three cars where two cars have the same number of people.

Table 4 *Collaboration of Three Cars with n, n, and* P People

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Volume-based allocation	Cost (€) pp after collaboration: Shapley value
Car 1 (*n*)	100/*n*	100/(2*n*+*p*)	(100-66.6)/*n*
Car 2 (*n*)	100/*n*	100/(2*n*+*p*)	(100 - 66.6)/*n*
Car 3 (*p*)	100/*p*	100/(2*n*+*p*)	(100 – 66.6)/*p*

Note that when n = 1 and p = 2, Table 4 is same as Table 2.

Collaboration 3: A collaboration of three cars with different number of in each car is presented below.

Table 5 *Collaboration of Three Cars with n, p, and Q People*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Volume-based allocation	Cost (€) pp after collaboration: Shapley value
Car 1 (n)	$100/n$	$100/(n+p+q)$	$(100\text{-}66.6)/n$
Car 2 (p)	$100/p$	$100/(n+p+q)$	$(100 - 66.6)/p$
Car 3 (q)	$100/q$	$100/(n+p+q)$	$(100 – 66.6)/q$

3. Carpooling with Mileage or wear and Tear Taken into Consideration

During this exercise in class, one of the teams literally treated this as an actual carpooling exercise. The discussion was about adding miles (and thus wear and tear), a breakdown or a possible collision to the car driven to the beach. A collision or breakdown will be most inconvenient for the car's owner in terms of time and money spent. One student said that if his car goes to the beach, then he is not in agreement with the results obtained by volume-based rule or Shapley value. The team factored in all the arguments and said that the car going to the beach must pay less. Following this post-exercise analysis, we developed two gain sharing rules for this carpooling scenario. We present these rules below and provide an explanation.

The Dummy Passenger Rule: **Step 1.** Add a *dummy* passenger to the car that is going to the beach; *Step 2.* Apply the Shapley value method for all cars (and passengers) involved in the collaboration; and *Step 3*. Transfer the dummy passenger's cost to the other cars (and passengers) by using the volume-based rule.

Assume that one yellow car and two white cars collaborate, and the yellow car goes to the beach and thus adds miles. Since Shapley value considers prior efficiencies, a dummy passenger is first added to the yellow car and then Shapley value is calculated. Table 6 (a) below displays the costs upon applying Steps 1 and 2.

Table 6 (a) *Collaboration of One Yellow Car (Adding Miles) & Two White Cars*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Shapley value
Yellow (2 + **1**)	50	(100 – 66.6)/**3** = **11.1**
White (1)	100	(100 – 66.6)/1 = 33.3
White (1)	100	(100 – 66.6)/1 = 33.3

Although 11.1***3**+33.3*1+33.3*1 = 99.9 ≈ €100, the third person in the yellow car is a non-existent dummy passenger and cannot pay €11.1. So, step 3 of this rule transfers this cost to the other cars by using the volume-based approach. Since there are two people in the two white cars combined, it will cost them an additional €11.1/2 or €5.55 each and thus a total of 33.3 + 5.55 = €38.85/person. We present these final costs in Table 6(b) below and also remove the dummy passenger from the yellow car.

Table 6 (b) *Collaboration of One Yellow Car (adding miles) and Two White Cars*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration Shapley value & volume-based rules
Yellow (2)	50	11.1
White (1)	100	38.85
White (1)	100	38.85

It is assumed that the 2nd person in the yellow car is a family member and hence does not pay extra. Comparing Table 6(b) with Table 2, it is clear that the Dummy Passenger Rule favors the yellow car. Since the white cars do not add miles, those parties are likely to absorb the extra €5.55/person. This is analogous to the case of renting a car versus driving one's own car. Clearly, there are infinite ways to collaborate in this scenario. For example, one could also argue that the yellow car must pay less or not pay at all.

We now provide an algebraic expression for this rule. Table 7 given below illustrates Steps 1 and 2. Step 3 of this Dummy Passenger Rule is provided below Table 7.

Table 7 *Collaboration of Car 1 (n), Car 2 (p) and Car 3 (q) where Car 1 adds Miles*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Shapley value
Car 1 (n + **1**)	100/n	(100 – 66.6)/(n + 1) = 33.3/(n + **1**)
Car 2 (p)	100/p	(100 – 66.6)/p = 33.3/p
Car 3 (q)	100/q	(100 – 66.6)/q = 33.3/q

As per Step 3, we transfer $33.3/(n+1)$ associated with the dummy passenger to Cars 2 and 3 using the volume-based approach. Thus, each person in Car 2 will pay $(33.3/p) + (33.3)/(n+1)*(p+q)$. And each person in Car 3 will pay $(33.3/q) + (33.3)/(n+1)*(p+q)$. To verify the accuracy of our allocation, we show that $[33.3/(n+1)]*n + [(33.3/p) + (33.3)/(n+1)*(p+q)]*p + [(33.3/q) + (33.3)/(n+1)*(p+q)]*q \Rightarrow 33.3n/(n+1) + [33.3 + 33.3p/(n+1)*(p + q)] + [33.3 + 33.3q/(n+1)*(p + q)] \Rightarrow [33.3n*(p+q) + 33.3*(n+1)*(p+q) + 33.3p + 33.3*(n+1)*(p+q) + 33.3q]/(n+1)*(p+q) \Rightarrow$
$[33.3np + 33.3nq + 66.6np + 66.6nq + 66.6p + 66.6q + 33.3p + 33.3q]/(n+1)*(p+q) \Rightarrow [99.9np + 99.9nq + 99.9p + 99.9q]/(n+1)*(p+q) \Rightarrow 99.9*(n+1)*(p+q)/(n+1)*(p+q) \Rightarrow 99.9 \approx 100$.

The final value is 99.9 because we approximated one-third of 200 to be 66.6 while finding Shapley values.

We now propose our second Rule for gain sharing in the case of an actual carpooling scenario.

The 50% Reduction Rule: **Step 1.** Apply Shapley value method for all cars (and passengers) involved in the collaboration; *Step 2*. Reduce the cost by 50% for the car to be driven to the beach; and *Step 3*. Make the other cars (and passengers) absorb this 50% cost by using the volume-based rule.

Assume one yellow car and two white cars collaborate and the yellow car goes to the beach. Apply Shapley value method and also reduce yellow car's cost by 50%. This results in Table 8 (a) as given below.

Table 8 (a) *Collaboration of One Yellow Car (adding miles) and Two White Cars*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Shapley value
Yellow (2)	50	50% of (100 – 66.6)/2 = 50% of 16.6 = 8.3
White (1)	100	(100 – 66.6)/1 = 33.3
White (1)	100	(100 – 66.6)/1 = 33.3

Note that 8.3*2 + 33.3*1 + 33.3*1 = €83.2. As per Step 3 of this Rule, the remaining €16.6 must be absorbed by the other cars using the volume-based rule. Thus, it will cost them an additional €16.6/2 = €8.3 each and thus a total of 33.3 + 8.3 = €41.6/person. We present final costs in Table 8 (b) below.

Table 8 (b) *Collaboration of One Yellow Car (adding miles) and Two White Cars*

Car (# of people)	**Cost (€) pp before collaboration**	**Cost (€) pp after collaboration: Shapley value & volume-based rules**
Yellow (2)	50	8.3
White (1)	100	41.6
White (1)	100	41.6

Note that the yellow car pays only €8.3/person here but pays €11.1/person for the Dummy Passenger Rule.

We now provide an algebraic expression for this rule. Table 9 given below illustrates Steps 1 and 2. Step 3 of this 50% Reduction Rule is provided below Table 9.

Table 9 *Collaboration of Car 1 (n), Car 2 (p) and Car 3 (q) Where Car 1 Adds Miles*

Car (# of people)	**Cost (€) pp before collaboration**	**Cost (€) pp after collaboration: Shapley value**
Car 1 (n)	100/n	50% of (100 – 66.6)/n = 16.6/n
Car 2 (p)	100/p	(100 – 66.6)/p = 33.3/p
Car 3 (q)	100/q	(100 – 66.6)/q = 33.3/q

Step 3 is to make Cars 2 and 3 absorb that 50% i.e., €16.6. So, Car 2 will pay $(33.3/p) + (16.6)/(p+q)$ per person. And Car 3 will pay $(33.3/q) + (16.6)/(p+q)$ per person. To verify the accuracy of our allocation, we show that $[16.6/n]*n + [(33.3/p) + (16.6)/(p+q)]*p + [(33.3/q) + (16.6)/(p+q)]*q \Rightarrow$

$[16.6 + 33.3 + (16.6p)/(p+q) + 33.3 + (16.6q)/(p+q)] \Rightarrow [83.2 + 16.6*(p+q)/(p+q)] \Rightarrow 99.8 \approx 100$.

We now prove that our two proposed rules are equivalent for a special case. We present this as a Remark.

Remark: The Dummy Passenger Rule is equivalent to the 50% Reduction Rule for any number of cars as long as the car going to the beach has only one person (i.e., $n = 1$) before collaboration.

Proof: Consider Car 1(n), Car 2(p), and Car 3(q) where Car 1(n) adds miles by going to the beach. As per Table 7 (for the Dummy Passenger Rule), the cost/person for the car going to the beach is $(33.3)/(n+1)$. Thus, total cost for n people = $33.3n/(n+1)$. As per Table 9 (for the 50% Reduction Rule), the cost/person for the car going to the beach is $16.6/n$ or $33.3/2n$ and the total cost is $33.3n/2n$. Thus, $33.3n/(n+1) = 33.3n/2n$ only if $n+1 = 2n$ i.e., if $n = 1$. This proves that the cost for Car 1 is same for both rules when $n = 1$. The cost of dummy passenger (Table 7) and the 50% reduction amount (Table 9) are both equal to 16.6. Since the same amount is absorbed by Cars 2 and 3 using the volume-based rule, the costs for Cars 2 and 3 are also same when $n = 1$.

4. Carpooling with a Frequent Traveler having free Parking Privilege

Like how the car going to the beach argues for lower costs due to wear and tear, the frequent traveler with free parking privileges also makes such an argument. A frequent traveler is someone who has purchased an annual membership to the beach and may have benefits such as free parking. Free parking implies that it will now cost less than €100 for a trip to the beach. And it is reasonable to assume that parking fee at popular destinations like beaches can even be €30 per car. So, when a passenger is able to obtain free parking, everyone in that car benefits. Hence, this free parking scenario can be taken into consideration during such a collaborative gain sharing exercise. We consider two cases here.

Case 1: The owner of the car going to the beach is also a frequent traveler with free parking privilege. This means that this car will negotiate for really low costs per person since both mileage as well as free parking for everyone are now factored into the negotiations. This can be calculated rather easily by using the earlier algorithm of the Dummy Passenger Rule but by adding “two” dummy passengers for the car being driven to the beach. This can be left as an exercise for the students. It is important to keep in mind that adding two dummy passengers is only one possible way to negotiate. Again, in theory, there are an infinite number of possible negotiating strategies.

Case 2: The passenger with the privilege of free parking is not taking his car to the beach. So, this scenario will have at least two passengers negotiating for lower costs. The passenger who is driving the car provides the wear and tear argument for lower trip costs and the passenger with free parking privilege also negotiates for a reduction of costs for the trip. We present the following rule for Case 2.

The Frequent Traveler Rule: This proposed rule is essentially a variation of the Dummy Passenger Rule. Step 1. Add a dummy passenger to the car that is to be driven to the beach; Step 2. Add a dummy passenger to the car with the frequent traveler. Step 3. Apply the Shapley value method for all cars (and passengers) involved in the collaboration; and *Step 4*. Transfer the two dummy passenger costs to the other cars (and passengers) by using the volume-based rule.

Assume that one yellow car and two white cars collaborate, and the yellow car goes to the beach and thus adds miles. And the frequent traveler is in one of the white cars. Since Shapley value considers prior efficiencies, a *dummy* passenger is first added to the yellow car and one of the white cars and then the Shapley value is calculated. Table 10 (a) below displays the costs upon applying Steps 1, 2, and 3.

Table 10 (a) *Collaboration of One Yellow Car (adding miles) & Two White Cars with a Frequent Traveler in one of the White Cars*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Shapley value
Yellow (2 + **1**)	50	(100 – 66.6)/**3** = **11.1**
White (1 + **1**)	100	(100 – 66.6)/**2** = **16.6**
White (1)	100	(100 – 66.6)/1 = 33.3

Although 11.1***3**+16.6*2+33.3*1 = 99.8 ≈ €100, the third person in the yellow car and the second person in one of the white cars are non-existent dummy passengers and cannot pay €11.1 and €16.6 respectively. So, Step 4 of this rule transfers this cost to the other white car by using the volume-based approach. Since there is only one person in the other white car, it will cost that passenger an additional €11.1 + €16.6 and thus a total of 33.3 + 11.1 + 16.6 = €61. We present these final costs in Table 10(b) below and also remove the dummy passengers from the yellow car and the white car.

Table 10 (b) *Collaboration of One Yellow Car (Adding Miles) and Two White Cars with a Frequent Traveler in one of the White Cars*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration Shapley value & volume-based rules
Yellow (2)	50	11.1
White (1)	100	16.6
White (1)	100	61

It is assumed that the 2nd person in the yellow car is a family member and hence does not pay extra.

Comparing Table 10(b) with Table 2, it is clear that the Dummy Passenger Rule favors the yellow car as well as the white car with the frequent traveler. Since the other white car neither adds miles nor provides free parking, it ends up paying a lot more. Again, there are infinite ways to collaborate in this scenario. For example, one may argue that the yellow car must not pay at all. One may also argue that only 0.5 dummy passengers must be added for the traveler with free parking privileges.

We now provide an algebraic expression for this rule. Table 11 given below illustrates Steps 1, 2, and 3. Step 4 of this Frequent Traveler Rule is provided below Table 11.

Table 11 *Collaboration of Car 1 (n), Car 2 (p) and Car 3 (q). Car 1 Adds Miles and Car 2 has a Frequent Traveler*

Car (# of people)	Cost (€) pp before collaboration	Cost (€) pp after collaboration: Shapley value
Car 1 ($n + \mathbf{1}$)	$100/n$	$(100 – 66.6)/(n + 1) = 33.3/(n + \mathbf{1})$
Car 2 ($p + \mathbf{1}$)	$100/p$	$(100 – 66.6)/(p + 1) = 33.3/(p + \mathbf{1})$
Car 3 (q)	$100/q$	$(100 – 66.6)/q = 33.3/q$

As per Step 4, we transfer $33.3/(n+1)$ associated with the dummy passenger from Car 1 and $33.3/(p+1)$ associated with the dummy passenger from Car 2 to the q passengers in Car 3 using the volume-based approach. Thus, each person in Car 3 will pay $(33.3/q) + (33.3)/(n+1)*q + (33.3)/(p+1)*q$. To verify the accuracy of our allocation, we show that $[33.3/(n+1)]*n + [33.3/(p+1)]*p + [(33.3/q) + (33.3)/(n+1)*q + (33.3)/(p+1)*q]*q \Rightarrow$

$33.3n/(n+1) + 33.3p/(p+1) + 33.3 + 33.3/(n+1) + 33.3/(p+1) \Rightarrow$

$[33.3n*(p+1) + 33.3p*(n+1) + 33.3*(n+1)*(p+1) + 33.3*(p+1) + 33.3*(n+1)]/(n+1)*(p+1) \Rightarrow$

$[33.3np+33.3n+33.3np+33.3p+33.3np+33.3p+33.3n+33.3+33.3p+33.3+33.3n+33.3]/(n+1)*(p+1) \Rightarrow$

$[99.9np + 99.9p + 99.9n + 99.9]/(n+1)*(p+1) \Rightarrow 99.9*(n+1)*(p+1)/(n+1)*(p+1) \Rightarrow 99.9 \approx 100.$

The final value is 99.9 because we approximated one-third of 200 to be 66.6 while finding Shapley values.

5. Conclusion

This paper was motivated by a recent article on collaborative gain sharing. We propose new algorithms for carpooling with the objective of reducing the cost for the car adding mileage as well as for the frequent traveler with free parking privileges. We provide algebraic expressions for these algorithms. Furthermore, we prove that our proposed rules are equivalent for a special case. In the case of the frequent traveler with free parking privileges, we propose an algorithm which is a variation of the Dummy Passenger Rule.

6. References

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7. Appendix

Going to the Beach Exercise

Problem Definition

Each of you represents a group of people travelling in different cars. You have found out that you are all going to the coast this weekend. You were all planning to go in several cars with parties of different sizes. You now decide instead that it would be wiser to carpool, in order to increase efficiency and reduce costs.

1. The cost of the trip is €100 per car, independent of the number of people it carries.
2. Each car is identical in size and can carry a maximum of five people. If a car is not used, the community saves €100.
3. The color of your sheet represents the color of the car. A red car indicates a party of five people going to the beach; a blue car indicates it carries a party of four (and has one empty seat); a green car has three people; yellow, two; and white, one.

Objective

You need to find a collaboration with another party and a solution to share the costs/gains that is the most convenient for each of you individually and collectively. Negotiate an agreement with other parties in order to decrease your own party costs.

4. You negotiate on behalf of everyone at your original car party. The final costs will be split evenly among the original party participants.
5. At the start of the game each party is in its own car and splits the cost evenly across occupants. The costs/person at the start of the game were thus €100 for a person traveling alone (white car); €50 for parties of two (yellow), €33.3 for three (green), €25 for four (blue), and €20 for five (red).

Guidelines

Calculate your costs before and after the agreements you make and calculate cost savings. You can split your party into more groups as long as you manage to decrease your overall costs. Remember whatever your finals costs are, you divide them equally across each person of your party. In general, your party must act as a rational economic being; that is, you cannot accept an offer that will lead your party to pay more than your initial or present condition.

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