Retailer’s Optimal Order Level for Deteriorating Items under the Effect of Preservation Technology and Inflation with Partial Trade Credit

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Retailers face many challenges in the dynamic retail environment while managing deteriorating items. Preservation technology plays a vital role in extending the shelf life of perishable products, mitigating deterioration rates, and reducing inventory costs. This study incorporates the effect of preservation technology on the deterioration rate and its impact on retailer decisions. Inflation affects the purchasing power of money over time, impacting inventory costs and revenue. For the optimal order level, a mathematical model is developed that considers the effect of preservation technology, inflation, and trade credit. A numerical illustration was conducted to validate the model. The results provide insights into the trade-offs between these factors and their implications for retailers’ inventory control decisions. The finding offers practical implications for retailers seeking to enhance their inventory decision strategies in a dynamic and competitive retail environment. Sensitivity analysis is conducted to explore the impact of various parameters. The analysis of this paper shows that a higher trade credit period causes the highest total profit.

Keywords: Deterioration, Partial Trade Credit, Preservation Technology Investment, Inflation, Green Technology Investment.

1. Introduction and Literature Review

In the dynamic landscape of retail operations, managing inventory efficiently is crucial for retailers to remain competitive and maximize profitability. One key challenge in the retail industry is effectively handling deteriorating items which are products that experience a loss of value over time. Deterioration can occur due to various factors such as expiration, obsolescence, or physical decay. Ghare and Schrader (1963) first developed mathematics for deteriorating items. Singh and Saxena (2013) developed inventory models for deteriorating items with flexible manufacturing and reverse logistics operations. Preservation technology plays an important role in mitigating the impact of deterioration on inventory. It encompasses techniques and methods employed to extend the shelf life or maintain the quality of products for a longer duration. By utilizing preservation technology, retailers can minimize the deterioration rate and preserve the value of their inventory. Hsu et al. (2010) first introduced the preservation technology concept in their work. Shen et al. (2019) developed an EPQ model for deteriorating items with collaborative preservation technology investment under the carbon tax. Md. et al. (2020) formulated an inventory model with preservation technology that considers non-instantaneous deterioration. They also studied the trade credit effect in their model. Sepehri et al. (2021) formulated a sustainable EPQ model that considers imperfect quality items. They also consider preservation technology and quality improvement investment. Bhawaria and Rathore (2022) developed a Production inventory model for deteriorating items under the effect of inflation by considering preservation technology and stock and price-dependent demand. Singh et al. (2022) developed an inventory model with the effect of preservation technology on different carbon emission policies. Gautam et al. (2023) developed a sustainable inventory model for retailing business. To moderate deterioration they use preservation technology.

Inflation is another critical factor that affects the optimal order level for deteriorating items. In today's scenario inflation is becoming common in every developing country like India therefore to make our model more realistic we have considered the factor of inflation. In layman's language, inflation is defined as the rise of prices. However for inventory managers, it is one of the important factors to maintain their inventory. Buzacott (1975) gives the concept of inflation in inventory modelling. Goyal and Giri (2003) determined the optimal replenishment policies for a production-inventory model with deteriorating items, stock-dependent demand, and partial backlogging under the effect of inflation. Hou (2006) developed an inventory model for deteriorating items with stock-dependent consumption rates and shortages under inflation and time discounting. Singh et al. (2010) contributed to an inventory model for deteriorating items with shortages and stock-dependent demand under inflation for two shops under one management. Singh et al. (2013) developed a model for imperfect quality items with the effect of learning and inflation for two warehouse systems. Kumar and Kumar (2016) discussed the impact of system parameters in an inflationary environment. Khanna et al. (2020) developed an inventory model under the effect of inflation. They considered pricing decisions for imperfect quality items with inspection errors, and sales returns. Yadav et al. (2023) Smart production system model for the imperfect process. They developed a model with partial backordering and deterioration in an inflationary environment.

Trade credit can play a significant role in the procurement of inventory. In practice, supplier frequently offers their customers a delay of a fixed time for selling the amount, usually, there is no interest charge if the outstanding amount is paid within the permissible period, however, if the payment is not paid in full by the end of permissible delay period; then interest is charged on the outstanding amount. Therefore, it is clear that a customer will delay the payment up to the last moment of the permissible period allowed by the supplier. Before the end of the trade credit period, the retailer sells the goods, accumulates revenue, and earns interest. Interest earned can be thought of as a return on investment since the money generated through revenue can plow back into the business. Therefore, it makes economic sense for the customer to delay the settlement of the replenishment account up to the last day of the credit period allowed by the supplier. Haley and Higgins (1973) were the first to introduce a studied trade credit financing policy for the economic order quantity model. Goyal (1985) developed an inventory model with permissible delay in payment. Chang et al. (2003), developed a model for deteriorating items under trade credit policy associated with order size. Singh (2008) developed a perishable inventory model with quadratic demand, partial backlogging, and permissible delay in payment. Tiwari et al. (2018) developed a sustainable green production model for many items under trade credit and shortages. Hasan et al. (2021) developed an innovative model of investment in green technology investment to reduce carbon emissions. Taleizadeh et al. (2022) developed an inventory model with partial backordering that took carbon emissions and trade credit policies into account. Alamri (2023) developed a sustainable inventory model with a trade credit policy.

Therefore, this study aims to explore the optimal order level for deteriorating items in the context of retail operations. It considers the combined effects of preservation technology, green technology, inflation, and trade credit to provide retailers with a comprehensive framework for inventory management. By accounting for these factors, retailers can make informed decisions that minimize costs, maximize profitability, and ensure optimal inventory levels for deteriorating items.

Selected Contributions:

We selected some contributions, tabulated in Table 1, as given below.

References	Deteriorating items	Preservation Technology	Demand Stock Dependent	Inflation	Trade credit
Kumar and Kumar (2016)	✓	✓	✓	✓	✓
Tiwari et al. (2018)	✓	✓	✓	✓	✓
Shen et al. (2019).	✓	✓	×	×	×
Md Mashud et al. (2020)	✓	✓	×	×	✓
Sepehri (2021)	✓	✓	×	×	×
Bhawaria and Rathore (2022)	✓	✓	✓	✓	✓
Yadav et al. (2023)	✓	✓	✓	✓	✓
Present Paper	✓	✓	✓	✓	✓

The outlines of the present paper are as follows: section 2 represents the notations and assumptions to design this economic order quantity model. In section 3, the optimization problem for the model is given. Also, the optimal solution is derived in this section. Various numerical examples are discussed in section 4. Section 5 provides sensitivity analysis for different parameters, managerial insights, and observations are discussed. Finally, the conclusion of this study is discussed in section 6.

2. Mathematical Model

Consider an economic order quantity model for sole retailers. The retailer orders a quantity q at the beginning of the period. The demand is taken as selling price dependent. Preservation technology is used to protect the products from deterioration. Effect of inflation is taken into account. In the time interval $[0, T]$, the inventory level $I(t)$ reduces to zero due to demand and deterioration.

The mathematical model is based on the following assumptions and notation.

2.1 Assumptions

Assumptions are made as follows.

- The demand rate is dependent on selling price

$$D(p) = a - bp$$

Where $a > b, p < \frac{a}{b}$, a, and b are constant parameters.

- This model contains a single item and single retailer.
- Lead time is negligible and replenishment rate is instantaneous.
- The inventory system has an infinite planning horizon.
- Effect of inflation is taken into consideration.
- Investment in green technology is taken into consideration.
- If the retailer orders a quantity q partial trade credit M is provided by the supplier. An amount (1-δ)cq is paid by the retailer on the receipt of the products and this time retailer has trade credit M for rest of the payment δcq.
- Investment in preservation technology (PT) is taken into consideration. The amount of reduced rate of deterioration after using investment in PT is $\lambda(\alpha) = ye^{-u\alpha}$, where this function satisfies the conditions $\lambda'(\alpha) > 0, \lambda''(\alpha) < 0 \ and \ \lambda(0) = y \ and \ 0 < u < 1$

2.2 Notation

O	**Ordering cost per order ($/ Per order)**
c	Purchasing cost per unit item (Rs./unit)
p	Selling price per unit item $p > c$ (Rs./unit)
D	Demand rate per unit time
a	Demand scale parameter (Unit)
b	Price sensitive parameter
ψ	Rate of holding cost for goods (in percentage) ($ / Unit)
G	Investment in green technologies to reduce carbon emissions ($)
I_e	The interest rate earned by the retailer (in %)
I_p	The interest rate paid by the retailer (in %)
q	Retailer's order quantity per cycle (Unit)
y	Initial deterioration rate
$\lambda(\alpha)$	Deterioration rate after using preservation technology ($)
u	The sensitive parameter in deterioration rate when using preservation technology
r	Rate of inflation
δ	Percentage of the payment which is permitted to be delayed when retailer order, $0 \leq \delta \leq 1$
$I_1(t)$	Inventory level during the time interval [0, T]
T	Replenishment cycle length (years)

This model is developed for any retailing business, in which the retailer order q unit quantity from the supplier. The behavior of the inventory level is shown in Figure 1.

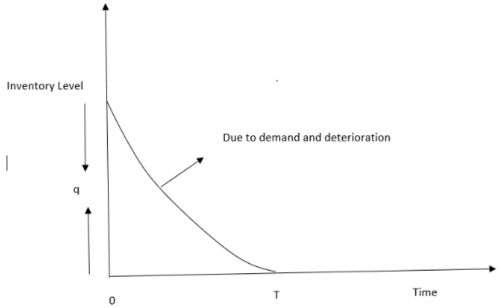

Figure 1 *Representation of Inventory Level*

The differential equation which represents the inventory level is given as follows.

$$\frac{dI_1(t)}{dt} + \lambda(\alpha)I_1(t) = -(a - bp),\ 0 \le t \le T \quad (1)$$

With boundary conditions $I_1(0) = q, I_1(T) = 0$.
After solving these equations (1) and (2) with the help of boundary conditions, we get

$$I_1(t) = \frac{(a-bp)}{\lambda(\alpha)}\left(e^{\lambda(\alpha)(T-t)} - 1\right),\ 0 \le t \le T \quad (2)$$

Ordered quantity for this model is

$$q = \frac{(a-bp)}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T} - 1\right]$$

The total profit for the retailer contains the following terms,
1. Ordering cost (OC) $= O$
2. Holding cost (HC) $= c\psi \int_0^T e^{-rt} I_1(t)dt$

$$= \frac{c\psi(a-bp)}{\lambda(\alpha)}\left(\frac{-e^{-rT} + e^{\lambda(\alpha)T}}{(\lambda(\alpha) + r)} + \frac{(e^{-rT} - 1)}{r}\right)$$

3. Deterioration cost (DC) $= c\lambda(\alpha) \int_0^T e^{-rt} I_1(t)dt$

$$= c(a-bp)\left(\frac{-e^{-rT} + e^{\lambda(\alpha)T}}{(\lambda(\alpha) + r)} + \frac{(e^{-rT} - 1)}{r}\right)$$

4. Green Technology Investment (GI) $= GT$
5. Preservation Technology investment = αT
6. Sales revenue (SR) $= pq$

$$= \frac{(a-bp)p}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right]$$

There are two subcases which arise based on trade credit period M. Description of these cases is given in the following Table 2.

Table 2 *Description Subcases of the Model*

Subcase (a): $0 \le M \le T$
Subcase (b): $0 \le T \le M$

Since the partial trade credit is offer by the supplier to the retailer. So, the retailer orders a quantity q, partial trade credit M is provided by the supplier. An amount $(1-\delta)cq$ is paid by the retailer on the receipt of the products and this time retailer has trade credit M for rest of the payment δcq.

Subcase (a): $0 \le M \le T$

In this subcase, the retailer gets trade credit M which is less than time period T. At the rate of I_p, retailer has to pay interest for time period [M, T]. The interest paid (IP) by the retailer per cycle is as follows.

$$IP_1 = cI_p\left\{\frac{(1-\delta)(a-bp)M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right]+\int_M^T e^{-rt}I_1(t)dt\right\}$$

$$= cI_p\left\{\frac{(1-\delta)(a-bp)M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right] + \frac{(a-bp)}{\lambda(\alpha)}\left[\frac{e^{\lambda(\alpha)[T-M]-rM}-e^{-rT}}{(\lambda(\alpha)+r)}+\frac{e^{-rT}-e^{-rM}}{r}\right]\right\}$$

In the same time period, the retailer earns interest for the time period [0, M] at rate of I_e . So, the interest earned (IE) by the retailer per cycle is as follows.

$$IE_1 = I_e p\left\{\int_0^M (a-bp)e^{-rt}t\,dt\right\}$$

$$= I_e p(a-bp)\left(\frac{1-rMe^{-rM}-e^{-rM}}{r^2}\right)$$

Subcase (b): $0 \le T \le M$

In this subcase, the retailer gets trade credit M which is greater than time period T. The retailer does not have to pay any interest in the time period [0, T]. So, the interest paid (IP) by the retailer per cycle is as follows.

$$IP_2 = cI_p\left\{\frac{(1-\delta)(a-bp)M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right.\right\}$$

In the same time period, the retailer earns interest for the time period [0, M] at rate of I_e. So, the interest earned (IE) by the retailer per cycle is as follows.

$$IE_2 = I_e p\left\{\int_0^T (a-bp)e^{-rt}tdt + \int_0^T (a-p)(M-)e^{-rt}dt\right\}$$

$$= I_e p(a-bp)\left(\frac{-Te^{-rt}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r} + \frac{1}{r}\right]\right)$$

Therefore, the total relevant profit $TP(p,T)$ per unit for case (i) is given as

$$TP(p,T) = \begin{cases} TP_1(p,T) = \frac{1}{T}(SR - OC - HC - DC - GT - PT - IP_1 + IE_1), 0 \le M \le T \\ TP_2(p,T) = \frac{1}{T}(SR - OC - HC - DC - GT - PT - IP_2 + IE_2), 0 \le T \le M \end{cases}$$

Where,

$$TP_1(p,T) = \frac{1}{T}\left\{\frac{(a-bp)p}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T} - 1\right] - O - \frac{c\psi(a-bp)}{\lambda(\alpha)}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - c(a-bp)\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - GT - \alpha T - cI_p\left\{\frac{(1-\delta)(a-bp)M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T} - 1\right] + \frac{(a-bp)}{\lambda(\alpha)}\left[\frac{e^{\lambda(\alpha)[T-M]-rM} - e^{-rT}}{(\lambda(\alpha)+r)} + \frac{e^{-rT} - e^{-rM}}{r}\right]\right\} + I_e p(a-bp)\left(\frac{1-rMe^{-rM} - e^{-rM}}{r^2}\right)\right\} \quad (3)$$

$$TP_2(p,T) = \frac{1}{T}\left\{\frac{(a-bp)p}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T} - 1\right] - O - \frac{c\psi(a-bp)}{\lambda(\alpha)}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - C_d(a-bp)\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - GT - \alpha T - cI_p\left\{\frac{(1-\delta)(a-bp)M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T} - 1\right]\right\} + I_e p(a-bp)\left(\frac{-Te^{-rT}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r} + \frac{1}{r}\right]\right)\right\} \quad (4)$$

3. Solution Techniques

To find the optimal solution of the present economic order quantity model, we will use the following Theorem.

Necessary Conditions for Optimal Solution

Theorem 3.1: When replenishment cycle length T is fixed, the total profit function $TP_2(p,T)$ is concave with respect to the selling price p.

Proof: The partial derivative of first and second order of profit function $TP_2(p,T)$ with respect to selling price p, are given as follows:

$$\frac{TP_2(p,T)}{\partial \mathrm{p}} = \frac{1}{T}\left\{\frac{(a-2bp)}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right] + \frac{c\psi \mathrm{b}}{\lambda(\alpha)}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) + C_d \mathrm{b}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) + cI_p\left\{\frac{(1-\delta)\mathrm{b}M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right]\right\} + I_e(a-2bp)\left(\frac{-Te^{-rT}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r}+\frac{1}{r}\right]\right)\right\} \tag{5}$$

and

$$\frac{\partial^2 TP_2(p,T)}{\partial p^2} = \frac{1}{T}\left\{\frac{-2\mathrm{b}}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right] - 2bI_e\left(\frac{-Te^{-rT}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r}+\frac{1}{r}\right]\right)\right\} < 0 \tag{6}$$

According to equation (6), the total profit function for retailer is concave**.** Hence, the proof of the theorem 3.1 completed.

Theorem 3.2: When the selling price p is fixed, the total profit function $TP_2(p,T)$ is concave with replenishment cycle length T.
Proof: The partial order derivative of first and second order of total profit function $TP_2(p,T)$ with respect to replenishment cycle length are given as follows

$$\frac{\partial TP_2(p,T)}{\partial \mathrm{T}} = \frac{-1}{T^2}\left\{\frac{(a-bp)p}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right] - O - \frac{c\psi(a-bp)}{\lambda(\alpha)}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - c(a-bp)\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - cI_p\left\{\frac{(1-\gamma)(a-bp)M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right]\right\} + I_e p(a-bp)\left(\frac{-Te^{-rT}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r}+\frac{1}{r}\right]\right)\right\} + \frac{(a-bp)p}{T}\left[e^{\lambda(\alpha)T}\right] - \frac{c\psi(a-bp)}{\lambda(\alpha)T}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{c(a-bp)}{T}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{cI_p\{(1-\gamma)e^{\lambda(\alpha)T}(a-bp)M\}}{T} + \frac{I_e p(a-bp)}{T}\left(Te^{-rT} - \frac{e^{-rT}}{r} + (M-T)e^{-rT} - e^{-rT}\right) \tag{7}$$

$$\frac{\partial^2 TP_2(p,T)}{\partial T^2} == \frac{2}{T^3}\left\{\frac{(a-bp)p}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right] - O - \frac{c\psi(a-bp)}{\lambda(\alpha)}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - c(a-bp)\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - cI_p\left\{\frac{(1-\gamma)(a-bp)M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right]\right\} + I_e p(a-bp)\left(\frac{-Te^{-rT}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r}+\frac{1}{r}\right]\right)\right\} + \frac{(a-bp)p}{T}\left[e^{\lambda(\alpha)T}\right] - \frac{c\psi(a-bp)}{\lambda(\alpha)T}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{c(a-bp)}{T}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{cI_p\{(1-\gamma)e^{\lambda(\alpha)T}(a-bp)M\}}{T} + \frac{I_e p(a-bp)}{T}\left(Te^{-rT} - \frac{e^{-rT}}{r} + (M-T)e^{-rT} - e^{-rT}\right) - \frac{(a-bp)p}{T^2}\left[e^{\lambda(\alpha)T}\right] + \frac{(a-bp)p\lambda(\alpha)}{T}\left[e^{\lambda(\alpha)T}\right] + \frac{c\psi(a-bp)}{\lambda(\alpha)T^2}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{c\psi(a-bp)}{\lambda(\alpha)T}\left(\frac{-r^2e^{-rT}+(\lambda(\alpha))^2e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + re^{-rT}\right) + \frac{c(a-bp)}{T^2}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{c(a-bp)}{T}\left(\frac{-r^2e^{-rT}+(\lambda(\alpha))^2e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + re^{-rT}\right) + \frac{cI_p\{(1-\gamma)e^{\lambda(\alpha)T}(a-bp)M\}}{T^2} -$$

$$\frac{cI_p\{(1-\gamma)e^{\lambda(\alpha)T}\lambda(\alpha)(a-bp)M\}}{T} - \frac{I_e p(a-bp)}{T^2}\left(Te^{-rT} - \frac{e^{-rT}}{r} + (M-T)e^{-rT} - e^{-rT}\right) + \frac{I_e p(a-bp)}{T}\left(-rTe^{-rT} - r(M-T)e^{-rT} + re^{-rT}\right) < 0 \quad (8)$$

Sufficient Conditions for Optimal Solution

Theorem 3.3: The optimal values of decision variables p $*$ and T^* that maximize the total profit function $TP_2(p,T)$exist and are unique.

Proof: The solution of the total profit function will be optimal if the corresponding Hessian matrix H of that function $TP_2(p,T)$is negative definite. This Hessian matrix is negative definite if all the eigenvalues of it are negative where all its diagonal entries are negative.

$$H = \begin{pmatrix} \frac{\partial^2 TP_2(p,T)}{\partial T^2} & \frac{\partial^2 TP_2(p,T)}{\partial T \partial p} \\ \frac{\partial^2 TP_2(p,T)}{\partial p \partial \mathrm{T}} & \frac{\partial^2 TP_2(p,T)}{\partial p^2} \end{pmatrix}$$

$$\frac{\partial^2 TP_2(p,T)}{\partial T^2} == \frac{2}{T^3}\left\{\frac{(a-bp)p}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T} - 1\right] - O - \frac{c\psi(a-bp)}{\lambda(\alpha)}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - c(a-bp)\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right) - cI_p\left\{\frac{(1-\gamma)(a-bp)M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T} - 1\right]\right\} + I_e p(a-bp)\left(\frac{-Te^{-rT}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r} + \frac{1}{r}\right]\right)\right\} + \frac{(a-bp)p}{T}\left[e^{\lambda(\alpha)T}\right] - \frac{c\psi(a-bp)}{\lambda(\alpha)T}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{c(a-bp)}{T}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{cI_p\{(1-\gamma)e^{\lambda(\alpha)T}(a-bp)M\}}{T} + \frac{I_e p(a-bp)}{T}\left(Te^{-rT} - \frac{e^{-rT}}{r} + (M-T)e^{-rT} - e^{-rT}\right) - \frac{(a-bp)p}{T^2}\left[e^{\lambda(\alpha)T}\right] + \frac{(a-bp)p\lambda(\alpha)}{T}\left[e^{\lambda(\alpha)T}\right] + \frac{c\psi(a-bp)}{\lambda(\alpha)T^2}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{c\psi(a-bp)}{\lambda(\alpha)T}\left(\frac{-r^2e^{-rT}+(\lambda(\alpha))^2e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + re^{-rT}\right) + \frac{c(a-bp)}{T^2}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) - \frac{c(a-bp)}{T}\left(\frac{-r^2e^{-rT}+(\lambda(\alpha))^2e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + re^{-rT}\right) + \frac{cI_p\{(1-\gamma)e^{\lambda(\alpha)T}(a-bp)M\}}{T^2} - \frac{cI_p\{(1-\gamma)e^{\lambda(\alpha)T}\lambda(\alpha)(a-bp)M\}}{T} - \frac{I_e p(a-bp)}{T^2}\left(Te^{-rT} - \frac{e^{-rT}}{r} + (M-T)e^{-rT} - e^{-rT}\right) + \frac{I_e p(a-bp)}{T}\left(-rTe^{-rT} - r(M-T)e^{-rT} + re^{-rT}\right)$$

$$\frac{\partial TP_2(p,T)}{\partial \mathrm{p}\,\partial \mathrm{T}} = \frac{-1}{T^2}\left\{\frac{(a-2bp)}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right] + \frac{c\psi \mathrm{b}}{\lambda(\alpha)}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right)\right.$$
$$+ cb\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right)$$
$$+ cI_p\left\{\frac{(1-\gamma)\mathrm{b}M}{\lambda(\alpha)}[e^{\lambda(\alpha)T}-1]\right\}$$
$$\left. + I_e(a-2bp)\left(\frac{-Te^{-rT}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r}+\frac{1}{r}\right]\right)\right\}$$
$$+ \frac{(a-2bp)}{T}\left[e^{\lambda(\alpha)T}\right] + \frac{c\psi \mathrm{b}}{\lambda(\alpha)T}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right)$$
$$+ \frac{cb}{T}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} - e^{-rT}\right) + \frac{cI_p\left\{(1-\gamma)e^{\lambda(\alpha)T}\mathrm{b}M\right\}}{T}$$
$$+ \frac{I_e(a-2bp)}{T}\left(Te^{-rT} - \frac{e^{-rT}}{r} + (M-T)e^{-rT} - e^{-rT}\right)$$

$$\frac{\partial TP_2(p,T)}{\partial \mathrm{T}\,\partial \mathrm{p}} = \frac{-1}{T^2}\left\{\frac{(a-2bp)}{\lambda(\alpha)}\left[\lambda e^{\lambda(\alpha)T}-1\right]\right.$$
$$+ \frac{c\psi \mathrm{b}}{\lambda(\alpha)}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right)$$
$$+ C_d\mathrm{b}\left(\frac{-e^{-rT}+e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + \frac{(e^{-rT}-1)}{r}\right)$$
$$+ cI_p\left\{\frac{(1-\delta)\mathrm{b}M}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right]\right\}$$
$$\left. + I_e(a-2bp)\left(-\frac{e^{-rT}}{r^2} + \frac{1}{r^2} + M\left[\frac{-e^{-rT}}{r}+\frac{1}{r}\right]\right)\right\}$$
$$+ \frac{(a-2bp)\lambda(\alpha)e^{\lambda(\alpha)T}}{\lambda(\alpha)T} + \frac{c\psi \mathrm{b}}{\lambda(\alpha)}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + e^{-rT}\right)$$
$$+ C_d\mathrm{b}\left(\frac{re^{-rT}+\lambda(\alpha)e^{\lambda(\alpha)T}}{(\lambda(\alpha)+r)} + e^{-rT}\right)$$
$$+ I_e(a-2bp)\left(e^{-rT} + \frac{e^{-rT}}{r} + (M+1)e^{-rT}\right)$$

$$\frac{\partial^2 TP_2(p,T)}{\partial p^2} = \frac{1}{T}\left\{\frac{-2\mathrm{b}}{\lambda(\alpha)}\left[e^{\lambda(\alpha)T}-1\right]\right.$$
$$\left. - 2bI_e\left(\frac{-Te^{-rT}}{r} - \frac{e^{-rT}}{r^2} + \frac{1}{r^2} + (M-T)\left[\frac{-e^{-rT}}{r}+\frac{1}{r}\right]\right)\right\}$$

4. Numerical Analysis

Some numerical examples are solved here using MATHEMATICA 12.0 to see the procedure of getting optimality. The optimum solution for each case is given in the following Table 3, in which subcase (b) gives maximum total profit.

Table 3 *Optimum Solution for each Case*

Case (i)	**M**	**T**	$\boldsymbol{p}$	**TP** $(\boldsymbol{p}, \mathbf{T})$
Subcase (a)	1.5 Months	1.256 years	4375.2	156330 Rs
Subcase (b)	3 Months	0.213378 years	5915.89	179943 Rs

Example 5.1: Here we discuss an example for subcase (a) of case (i). The trade credit period M is less than the time period T. We are taking the values of parameters as $c = 450$ \$, $a = 100$, $b = 0.01$, $O = 100\$$, $\theta = 0.01$, $\beta = 0.1$, $b = 0.001$, $r = 0.022\%$, $\delta = 0.3$, $G = 40\$$, $\psi = 0.0025\$$, $y = 0.01$, $\alpha = 0.9\$$, $I_p = 0.09\%$, $I_e = 0.028\%$, and $u = 0.03$ then we get the optimal results as follows.

The relevant total profit for these values of parameters is $156330Rs$, optimal Trade Credit Period is $M = 1.5\ Months$, and the optimal length of replenishment cycle is, $T = 2.123104\ year$, $q = 6.553\ Unit$, p= 4375.2 Rs.

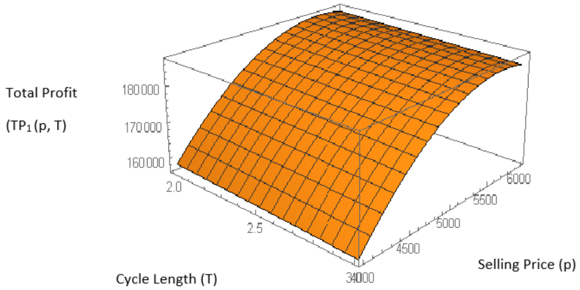

Figure 2 *Concavity of $TP_1(p, T)$ with Respect to p and T*

Example 5.2: Numerical for subcase (b) is discussed here. In this subcase, the credit period M is greater than T. We are taking the values of parameters as $c = 450Rs$, $a = 100$, $b = 0.01$, $O = 100$, $\theta = 0.01$, , $b = 0.001$, $r = 0.022, i.e.,$2.2%, $\delta = 0.3$, $G = 40\$$, $\psi = 0.0025\$$, $y = 0.001$, $\alpha = 0.9\$$, $I_p = 0.09$ i.e., 9%, $I_e = 0.028\ i.e.,$ 2.8%, and $u = 0.036$, $M = 3$ Months then we get the optimal results as follows.

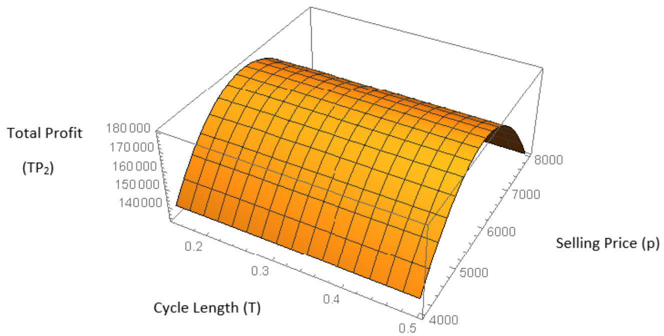

Figure 3 *Concavity of $TP_2(p, T)$ with Respect to p and T*

The relevant total profit for these values of parameters is $179943.00Rs$, optimal selling price is $p = 5915.89Rs$, and the optimal length of replenishment cycle is $T = 0.213378 year\ i.e., 2.5\ months$ and optimal order quantity $q = 8.71485\ Unit$.

5. Sensitivity Analysis

The sensitivity analysis of the various parameters is given in this section which shows the effect of change in parameters on the optimal solution. The sensitivity analysis is given in the following Table 4.

Table 4 *Sensitivity Analysis of Various Parameters*

Parameter	Values	T*	p^*	q^*	TP $(p*, T^*)$
c	500	0.215287	6017.67	8.57369	171062
	475	0.214321	5966.78	8.64429	175475
	425	Fail to	converge	solution	---------
	400	0.211558	5814.12	8.85582	189048
ψ	0.00255	0.21347	5918.03	8.71404	179754
	0.00253	0.213433	5917.18	8.71434	179830
	0.00251	0.213396	5916.32	8.71466	179905
	0.00249	0.213359	5915.46	8.71499	179981
r	0.0264	0.213621	5915.8	8.72496	179944
	0.0242	0.213443	5915.89	8.7175	179943
	0.0198	0.213312	5915.89	8.71215	179943
	0.0176	0.213182	5915.9	8.70682	179942
a	110	0.192575	6415.77	6.90251	226837
	108	0.196402	6315.79	7.23606	217025
	106	0.200385	6215.81	7.58316	207429
	104	0.206676	6065.85	8.13117	193442
G	48	0.213378	5915.89	8.71485	179935
	44	0.213378	5915.89	8.71485	179939
	36	0.213378	5915.89	8.71485	179947
	32	0.213378	5915.89	8.71485	179951
α	0.9	0.213378	5915.89	8.71485	179951
	0.8	0.213518	5915.5	8.7214	179986
	0.7	0.213659	5915.12	8.72797	180021
	0.6	0.213801	5914.74	8.73458	180055
A	115	0.228865	5915.98	9.35725	179883
	105	0.218661	5915.92	8.93975	179928
	95	0.206861	5915.85	8.45698	179980
	85	0.196685	5915.79	8.04068	180024
u	0.039	0.213273	5916.18	8.71868	179925
	0.036	0.213378	5915.89	8.7236	179951
	0.033	0.213483	5915.6	8.72852	179977
	0.03	0.213588	5915.31	8.73343	180003
I_e	0.0340	0.183589	5900.64	7.53267	184202
	0.0338	0.184062	5901.12	7.55121	184062
	0.0336	0.185210	5901.61	7.59744	183922
	0.0334	0.187723	5903.06	7.69789	183501
I_p	0.095	0.213271	5937.17	8.67379	178076
	0.09	0.213588	5915.31	8.73343	180003
	0.089	0.213653	5910.94	8.74278	180390
	0.083	0.214051	5884.71	8.79886	182718

6. Observations

In the present paper, we studied the economic order quantity model to check the optimality of total profit and replenishment cycle length. Some observations from the sensitivity analysis are as follows.

1. If the purchasing price (c) of items increase, the order quantity (q) decreases. Also, the increment in purchasing price, the total profit $TP_2(p,T)$ decreases while the selling price and cycle length (T) increases. It means that manager should avoid to increase the purchasing cost.
2. If the rate of holding cost of goods '*'ψ'*' increase, cycle length (T) increases while order quantity (q) and total profit $TP_2(p,T)$decreases. While the selling price slightly increases.
3. If the inflation rate increase order quantity (q), cycle length (T), and total profit $TP_2(p,T)$increases. While the selling price slightly decreases.
4. On increase in the demand scale parameter (a), the total profit $TP_2(p,T)$, and selling price (p) increases. While cycle length (T) and order quantity (q) decreases.
5. On increases in the investment in green technology (G) the total profit $TP_2(p,T)$slightly decreases. While cycle length (T), selling price, and order quantity (q) remain same.
6. On increases in the preservation technology investment ('α') order quantity (q), cycle length (T) and total profit $TP_2(p,T)$ decreases. While selling price increases.
7. On increase in the order cost 'A' the total profit $TP_2(p,T)$decreases. While cycle length (T), selling price (p) and order quantity (q) increases.
8. On increases in the deterioration parameter 'u' cycle length, order quantity and total profit $TP_2(p,T)$ decreases while selling price increases.
9. On increases in the rate of interest earned I_e the total profit $TP_2(p,T)$increasing. While cycle length (T), selling price and order quantity (q) decreases.
10. On increase in the interest paid rate total profit $TP_2(p,T)$, cycle length (T), and order quantity decreases. While the selling increases.

Managerial Insights

We observed some managerial insights for any industry which are as follows.

1. If the holding costs increase, the retailer should keep the cycle length minimum. As the holding costs are high, the total profit will decrease. Thus, to increase the total profit, holding the inventory for a shorter time is feasible.
2. To overcome the carbon emission manager should invest in green technology.
3. It is recommended that retailers should enhance the scale parameters in demand to increase the total profit.
4. To optimize the order levels, retailers must carefully consider the inflation rate and incorporate it into their decision-making process to ensure cost-effective inventory management.
5. Maximum the trade credit period profit will be maximum.

7. Conclusion

This paper presents an economic order quantity system for a retailer in which demand is selling price dependent, preservation technology investment, green technology investment, partial trade credit under the effect of inflation considered. In present study, the supplier offers partial trade credit to the retailer. Two cases are developed in the present study based on the trade credit period. Two numerical examples are discussed here to illustrate the optimal policies and scenarios of this economic order quantity model under all assumed cases help the retailer optimal decision. From the numerical results we conclude that we get the maximum profit when the trade credit period is greater than the replenishment cycle length. From the sensitivity analysis we conclude that the higher the investment in the preservation technology higher will be the profit. A sensitivity analysis of different parameters is presented to distinguish the behavioral patterns of parameters on retailer's policy for per unit profit.

The future research direction for the present study includes the extension of it to the production model, changing trade credit policy for different levels and using different carbon emission policies.

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