Inventory Models with Trade Credit Under Two Types of Customers

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Retailers' offering trade credit terms to customer can be regarded as a type of price reduction. In today's ever competitive marketplace, offering delay in payments or trade credits has become a commonly adopted method to retailers. Most of the inventory models with trade credit period assumed that the nature of customers is same. In this paper we considered as retailer classifies his/ her customers into two categories: reliable customers and general customers. Reliable customers are given trade credits and general customers will pay at the time of purchase and no trade credit is given. Though trade credit is given to the reliable customers they may not make their payment so promptly. As far as possible they will pay within the trade credit period. However, every time they may not pay within trade credit and in such cases they pay interest or penalty for the late payments. A probability distribution is considered for the pattern of payments of the customers. Retailer's problem is to find an optimal policy that maximizes his/ her profit. Illustration is given through numerical example.

Keywords: Inventory Model, Discounted Cash Flow, Trade Credit, Probability Distribution, Profit Function, Expected Net Present Value

1. Introduction

Number of alternatives under trade credits which are practiced have been discussed in the literature. Most of the models discussed earlier assumed that the supplier would offer the retailer a delay period, but the retailer would not offer the trade credit period to his/her customers. There are some situations where a retailer gives similar concession to a set of customers whom we can call as reliable customers. Other customers are called as general customers. Teng (2009) discussed a two-level trade credit policy offered in two types of customers who are categorized as good credit customers and bad credit customers. Good credit customers are given full trade credits without the collateral deposit. An economic order quantity (EOQ) model is established for a retailer who receives a full trade credit by its supplier and offers a partial trade credit to its bad credit customer and full trade credit to its good credit customers. Retailer frequently offers a partial down-stream trade credit to its credit risk customers who must pay a portion of the purchase amount at the time of placing an order as a security deposit in order to reduce credit risk, and then receive a permissible delay on the rest of the outstanding amount. The class of customers discussed in this article is not similar to this but based on what is practiced by merchants in good faith to keep the reliable customers as their permanent customers. Another variation that is incorporated in this paper is the idea of uncertainty in payment patterns under trade credit. In the literature it is found that there will not be any outstanding payment. Every time payments are made exactly at one point by all. This will not happen in practice.

2. Literature Review

Literatures on trade credits have considered various aspects in the last five decades. Several practical situations are considered. However, one of the assumptions made was that the customers are assumed to follow single rule of making the payment at stipulated time points. Numbers of alternatives trade credits which are practiced have been discussed in the literature. Haley and Higgins (1973) examined the effect of trade credit policy with the optimal inventory policy. However, Goyal (1985) is first to derive an optimal EOQ model under the condition of trade credit taking into interest charges and interest earned. Aggarwal and Jaggi (1995) obtained ordering policies of deteriorating items under permissible delay in payments. Jamal et al. (1997) further worked out for this model by allowing shortages. Teng (2002) revised Goyal (1985) conclusion by mathematically proving that it is economical for buyer have smaller order sizes whenever permissible delay in payments in force.

When supplier allows a fixed credit period to the retailer and retailer offers some trade credit to his customers it is known as two level of trade credit. Salameh et al. (2003) examined policies when a retailer can pay for the goods immediately upon the receipt of the order or delay the payment till the next replenishment order, where the supplier will charge interest over the delayed period. Yung-Fu (2003) modified this to include two levels of trade credit policy that is retailer receives trade credit and trade credit is provided to the customer to stimulate more demand. Huang (2007) further examined the retailers' replenishment decisions under two levels of trade credit policy when supplier gives trade credit only if order quantity is smaller than a fixed quantity. Chung and Liao (2004) gave lot-sizing decisions when trade credit policies depending on the ordering quantity. That is, when the order quantity is less than a permitted limit the payment for the product must be made immediately. Otherwise, the fixed trade credit period is permitted.

In inventory policies under trade credit for long term will consist of multiple payments at different time points. Discounted cash flow (DCF) approach tries to evaluate the values of these transactions as on today. Hence, different policies can be compared. Chung (1989) presented the DCF approach for the analysis of an optimal inventory policy in the presence of trade credit. Huang and Chung (2003) extended Goyal's (1985) model that includes the situation of cash discount offered by the supplier for immediate payment. Teng (2006) adopted discount DCF to establish the optimal order policies when suppliers offer a cash discount and payment delay at the same time. Chung and Liao (2006) incorporated all the concepts of a DCF approach and trade credits linked to ordering quantity and developed new model for deteriorating items. Hill and Pakkala (2005) discussed a DCF model associated with payments to suppliers and revenue streams from customer's demands under base stock systems. Tripathi and Misra (2010) developed EOQ model credit financing in economic ordering policies of non-deteriorating items with time-dependent demand rate in the presence of trade credit using a DCF approach. Some of the interesting and relevant papers related to trade credits by using the DCF approach for deteriorating rates and integrated inventory models are Ouyang *et al.* (2009), (2015), Jinn-Tsair Teng (2014) and Pin-Shou Ting (2015).

When retailers deal with customers it may not be practical in business to give trade credits to all customers. Teng (2009) categorized customers into two types namely, good customers and bad customers according to credit risk of the customers. The idea is to reduce credit risk while offering trade credit to customers. Teng (2009) discussed about a two-level trade credit policy to involving these two types of customers. Full trade credit is offered to good credit customers and partial trade credit to bad customers. That is, a collateral deposit is collected from credit risk customers on the purchase amount at the time of placing an order.

3. Model Development

This article takes into consideration two prominent practices followed in business. They are (i) customers are classified as reliable and general; and (ii) policy of payment made at stipulated time point under trade credit not strictly satisfied always. The classification of customers is mainly based on the trade credit condition. There are some situations where a retailer offers, trade credit to a set of customers whom we can call as reliable customers. One who is known to the retailer and regularly does business with the retailer and very prompt in making the payments comes under reliable customers to whom trade credit is offered. The retailer may give trade credits to reliable customers to maintain the business. General customers are those whom trade credit cannot be given as they are either not very well known to the retailer or they are not credit worthy. The second variation is necessary as it is not possible for all customers to pay every time exactly at a stipulated time, though reliable customers are supposed to pay within in stipulated trade credit period. However, a trend of payment by reliable customers can be determined similar to the payment made by retailer under trade credit as discussed in Mohini and Pakkala (2016). Usually, interest is not charged for the outstanding amount if it is paid with in the trade credit period. The delay in payment produces benefits to the customers. This policy may attract new customers who consider it as a type of price reduction. However, if the payment is not paid within the permissible delay period, then interest is charged on the outstanding amount. General tendency of making payment by the customer will be at the last day of the trade credit period rather than with in the period. This is justified because of time value of money. Most prominently it is necessary to consider the effects of the time value of money on the inventory system.

Though trade credit is given to the reliable customers they may not always strictly adhere to this even if it is optimal and their payment pattern could vary according to customers financial conditions from time to time. As far as possible, they will pay within the trade credit period. However, every time they may not pay within the trade credit and in such cases, they pay interest or penalty for the late payments. Here we consider different situations of a customer who will pay against delivery, sometimes, makes a payment within or at the trade credit limit. Also, in extreme situations the retailer may not be able to pay within the trade credit period. Though it may not happen regularly, but is not unlikely. All these situations are captured in the model through an appropriate probability distribution. Thus, the probability distribution is considered for the pattern of payments of all the reliable customers of the retailer. Additionally, late payment penalties are categorized into two cases, one is late payment penalty from the trade credit limit and other is late payment penalty from the purchased date. Both situations are considered separately and discussed in this paper.

To study the impact of transaction done by reliable customer and general customer, we observed that the revenue of the retailer is affected by the pattern of payments made by customers. Hence it is necessary to consider the inventory model as a profit maximization problem to determine the retailer's optimal inventory cycle time and optimal payment time under the trade credit policy.

As we discussed above, most of the time, making payment by the customer will be at the last day of the trade credit period rather than within the period. The cash transactions taking place in several cycles in the planning horizon have to be compared with reference to a single time point. Hence it is necessary to consider the effects of the time value of money on the inventory system. Thus, the DCF approach permits a proper recognition of the financial implication of the optimum order quantity, replenishment time, and total profit in inventory analysis. The mathematical model is determined for retailers' optimum inventory cycle time and optimal order quantity by taking the uncertainty in the payment pattern into the model. Furthermore, we provide an easy-to-use closed form approximate optimal solution to the problem for any given set of parameters. Numerical examples are given to investigate the effect of change in some parameter values of the optimal solution.

3.1 Notations and Assumptions

We shall use following notations and assumptions to build up the mathematical model of the problem under consideration.

3.1.1 Notations

D = Demand rate per year
A = Ordering cost per replenishment
C = Purchasing cost per unit item
h = Unit stock holding cost per item per year excluding interest charges
M = Trade credit period
T = Cycle time in years
r = interest rate or opportunity cost
r_p= Penalty rate for late payment, where $r_p > r$
T* = the optimal cycle time
Q* = the optimal order quantity = DT*
g(.) = Density function of the random duration of the payment after the trade credit (*Y*) whenever payment is made after the trade credit period
$M_Y(y)$ = Moment generating function of random variable *Y*.

3.1.2 Assumptions

1. Demand rate "D" is known and constant.
2. Shortages are not allowed.
3. Planning horizon is infinite.

4. Replenishment occurs instantaneously on ordering, that is, lead time is zero.
5. A DCF approach is used to evaluate the costs incurred at various time points.
6. The proportions of reliable customer and general customers respectively are ξ and $1-\xi$.

3.2 Two Models

As usual, one of the models describes policies under trade credit given by the retailer to reliable customer is irrespective of the replenishment cycle period. Thus, in this case we discuss the case of the trade credit period as independent of the replenishment cycle period. We will refer this model as Model A.

Trade credit offered by the retailer to reliable customer may or may not coincide with the replenishment period. Normally, the optimal replenishment cycle will not coincide with trade credit duration given by the retailer (Model A). However, the retailer may insist the trade credit duration equal to the optimal replenishment period as a condition so that he/she can have sufficient money at the time of replenishment to pay the wholesaler. This will improve the cash flow condition of the retailer. Hence, the retailer may give a deadline for the payment as the replenishment cycle period to the customer. The model that discusses the trade credit period that coincides with the replenishment cycle period is called as Model B. In this paper, we discuss both the situations and their mathematical models have been derived.

In the earlier work of Mohini Kumari and TPM Pakkala (2012) and (2016) have considered that supplier offers trade credit to retailer. Whereas in this paper, it is considered that the retailer gives trade credit to the reliable customers.

4. Methodologies

4.1 Model A: The Trade Credit Period is Independent of the Replenishment Cycle Period

First, the sales revenue due to reliable customers is discussed. In this model, the retailer gives target for the payment to reliable customers up to the trade credit limit which is independent of the replenishment cycle period. Some of the reliable customers may pay at the time of purchase; some are more inclined to pay within the trade credit limit. Also, there are situations where the reliable customers are not able to pay within the trade credit limit. The probabilities for all the situation is determined by the trend that is based on the past payment pattern of reliable customers. The probability that payment made at the time of purchasing an item (u=0) is p_1, probability that payment made within the trade credit period is p_2. The payment pattern made within trade credit period could be described by different probability distribution depending on the pattern of payments. One of the appropriate distributions is the uniform random variable. It is assumed that among those who pay within trade credit make payment according to a uniform distribution over (0. M] and probability that the payment made after the trade credit period with penalty is p_3. Let r_p be the rate of penalty for late payment beyond trade credit. Given that payment delay is beyond trade credit, the excess delay in time '*u-M*' is assumed to follow Gamma distribution.

The two components of profit function are the sales revenue and the total cost which are separately obtained in the following sections.

4.1.1 The Expected Net Present Value of Sales Revenue for the Reliable Customers

The net present value of sales revenue per unit item by the reliable customers at various time points of payment when demand occurs at t in the nth period, ($0 \le t \le T$), includes three components

1. Sales revenue of all future cash flow when the payment is made at the time of purchasing an item (u=0) with probability p_1 ,
2. Sales revenue of all future cash flow when the payment is made at time u, within the trade credit period, where $0 < u \le M$. This is with a probability p_2. Payment during this period could occur at any random time point and assume payment time during this interval follows uniform random variable say 'u'. i.e., 'u' follows a uniform distribution (0, M].
3. Sales revenue of all future cash flow when payment made after the trade credit limit is with probability p_3. Here penalty rate r_p is charged from the time of trade credit period.

$$\begin{cases} Se^{-r(t+(n-1)T)} & u=0 \\ Se^{-r(t+(n\text{-}1)T+u)} & 0<u\le M \\ Se^{r_p(u-M)}e^{-r(t+(n\text{-}1)T+u)} & u\ge M \end{cases}$$

Hence expected net present value of sales revenue for nth period is

$$\int_0^T SD\left(p_1\, e^{-r\,(t+(n\text{-}1)T)} + p_2 \int_0^M e^{-r\,(t+(n\text{-}1)T+u)}\, f(u)\, du + p_3 \int_M^\infty e^{-r(t+(n\text{-}1)T+u)} e^{r_p(u-M)} g(u-M) du \right) dt$$

This can be split into three components.

Expected net present value of sales revenue for nth period when payment made at u=0 is

$$\int_0^T SDe^{-r\,(t+(\,n\text{-}1)T)}dt$$

$$= SDe^{-r\,(n\text{-}1)\,T}\,\frac{\left(1-e^{-rT}\right)}{r}$$

The expected net present value of the sales revenue for all the cycles in the planning

horizon when payment made at u=0 is $SD\,\dfrac{\left(1-e^{-rT}\right)}{r}\displaystyle\sum_1^\infty e^{-r\,(n\text{-}1)T} = \dfrac{SD}{r}$ (1)

Expected net present value of sales revenue per n^th^ period when payment is made within the trade credit limit is

$$\int_0^T \int_0^M SDe^{-r(t+(n\text{-}1)T+u)} f(u)\, du\, dt$$

$$= \int_0^T \frac{1}{M} \int_0^M SDe^{-r(t+(n\text{-}1)T+u))}\, du\, dt$$

$$= \int_0^T \frac{1}{M} SDe^{-r(t+(n\text{-}1)T)} \left(\int_0^M e^{-ru} du \right) dt$$

$$= \frac{SDe^{-r(n\text{-}1)T}\left(1-e^{-rT}\right)}{Mr^2}\left(1-e^{-rM}\right)$$

The expected net present value of the sales revenue for all the cycles in the planning horizon when payment is made within the trade credit limit is

$$\sum_1^\infty \frac{SDe^{-r(n\text{-}1)T}}{Mr^2}\left(1-e^{-rM}\right)\left(1-e^{-rT}\right)$$

$$= \frac{SD}{Mr^2}\left(1-e^{-rM}\right) \tag{2}$$

The expected net present value of sales revenue per n^th^ period when payment made after the trade credit period with penalty rate r_p is (Penalty is charged from the trade credit limit)

$$\int_0^T \int_M^\infty SDe^{-r(t+(n\text{-}1)T+u)} e^{r_p(u-M)} g(u-M) \quad du\, dt \tag{3}$$

$$= SDe^{-rM} e^{-r(n\text{-}1)T} \int_0^T e^{-rt} \int_0^\infty e^{-rv} e^{r_p v} g(v)\, dv\, dt$$

$$= SDe^{-rM} e^{-r(n\text{-}1)T} \int_0^T e^{-rt}\ M_Y(r_p - r)\, dt\ , Where \quad M_Y(t) = \int_0^\infty e^{tv} g(v)\, dv \tag{4}$$

Here $M_Y(t)$ represents Moment Generating Function (MGF) of the random variable for the duration of the payment made after the trade credit limit.
Any distribution which has nonnegative support can be considered for payment time beyond trade credit period. One of the appropriate distributions for the delay in payment beyond trade credit period is gamma distribution. In our discussion here, we assume this distribution and derive the revenue function.

MGF of Gamma distribution with parameter $(\alpha, \frac{1}{\theta})$ is $\dfrac{1}{\left[1-\dfrac{(r_p-r)}{\theta}\right]^{\alpha}}$ (5)

Substituting the expression (5), in (4) we get,

$$SDe^{-rM}e^{-r(n-1)T}\frac{(1-e^{-rT})}{r}\frac{\theta^{\alpha}}{(\theta+r-r_p)^{\alpha}}$$

The expected net present value of the sales revenue for all the cycles in the planning horizon when payment made after the trade credit period with penalty rate r_p is

$$\sum_{1}^{\infty}SDe^{-rM}e^{-r(n-1)T}\frac{\theta^{\alpha}(1-e^{-rT})}{r(\theta+r-r_p)^{\alpha}}=\frac{SDe^{-rM}\theta^{\alpha}}{r(\theta+r-r_p)^{\alpha}} \tag{6}$$

By using expressions (1), (2) and (6), expected net present value of the sales revenue for all the cycles in the planning horizon for different time periods with assigned probabilities p_1, p_2, and p_3 respectively is

$$\frac{SD}{r}p_1+\frac{SD}{Mr^2}\left(1-e^{-rM}\right)p_2+\frac{SDe^{-rM}\theta^{\alpha}}{r(\theta+r-r_p)^{\alpha}}p_3 \tag{7}$$

Next, sales revenue due to general customers is discussed. The expected net present value of the sales revenue for all the cycles in the planning horizon of general customer is same as equation (1) shown above, *i.e.*, payment made during the time of delivering the items

$$.\quad\int_{0}^{T}SD\sum_{1}^{\infty}e^{-r(t+(n-1)T)}dt=\frac{SD}{r} \tag{8}$$

Using the expression (7) and (8) the net expected present value of the sales revenue for different time points when reliable as well as general customers considered together with probabilities ξ and (1- ξ) respectively is

$$\left\{ \xi \left[\frac{SD}{r} p_1 + \frac{SD}{Mr^2}\left(1-e^{-rM}\right) p_2 + \frac{SDe^{-rM}\theta^{\alpha}}{r(\theta + r - r_p)^{\alpha}} p_3 \right] + (1-\xi)\frac{SD}{r} \right\} \tag{9}$$

4.1.2 Cost Components

The present value of the total cost is the sum of the ordering cost, purchase cost and inventory carrying cost. The expression as already derived in Mohini and Pakkala (2012)

The present value of the ordering cost for all the cycles in the planning horizon is

$$\frac{A}{(1-e^{-rT})}.$$

The present value of the purchase cost for all the cycles in the planning horizon is

$$\frac{CDT}{(1-e^{-rT})}$$

The present value of the inventory carrying cost for all the cycles in the planning horizon is

$$\frac{hDT}{r\left(1-e^{-rT}\right)} - \frac{hD}{r^2}$$

Hence, the present value of the total cost in the planning horizon is

$$\frac{A}{(1-e^{-rT})} + \frac{CDT}{(1-e^{-rT})} + \frac{hDT}{r(1-e^{-rT})} - \frac{hD}{r^2} \tag{10}$$

4.1.3 Profit Function and Optimal Replenishment Cycle Time

Using the expression (9) and (10), we express profit function as the difference between sales revenue and total cost

$$\begin{aligned} Z_1(T) = {} & \xi\left(\frac{SD}{r} p_1 + \frac{SD}{Mr^2}\left(1-e^{-rM}\right) p_2 + \frac{SDe^{-rM}\theta^{\alpha}}{r\left(\theta + r - r_p\right)^{\alpha}} p_3 \right) \\ & + (1-\xi)\frac{SD}{r} - \left(\frac{A}{(1-e^{-rT})} + \frac{CDT}{(1-e^{-rT})} + \frac{hDT}{r(1-e^{-rT})} - \frac{hD}{r^2} \right) \end{aligned} \tag{11}$$

To obtain the optimal value of T, we maximize the profit function Z_1 (T), at the optimal value of T the profit function has zero derivative and second order derivative is negative.
Which means the condition to maximize Z_1 (T) are

$$\frac{dZ_1(T)}{dT} = 0, \text{ and } \frac{d^2 Z_1(T)}{dT^2} \leq 0 .$$

$$\frac{dZ_1(T)}{dT} = 0 \Rightarrow \quad \frac{d}{dT}\left\{-\left(\frac{A}{(1-e^{-rT})} + \frac{CDT}{(1-e^{-rT})} + \frac{hDT}{r(1-e^{-rT})}\right)\right\} = 0$$

$$\frac{-Are^{-rT}}{(1-e^{-rT})^2} + \frac{CD\left(1-e^{-rT} - rTe^{-rT}\right)}{(1-e^{-rT})^2} + \frac{hD}{r}\frac{(1-e^{-rT} - rTe^{-rT})}{(1-e^{-rT})^2} = 0$$

After simplifications,

$$Ar - \left(CD + \frac{hD}{r}\right)(e^{rT} - rT - 1) = 0 \tag{12}$$

Also, we get $\dfrac{d^2 Z_1(T)}{dT^2} \leq 0$

Hence the optimal order size and optimal cycle time are derived by solving the equation (12).
Substituting the resultant value of T in the equation (11), we obtain the total optimum profit value.

4.1.4 An Approximation Method.

In order to give an approximate closed form solution to the above equation (12) we use a Taylor's series approximation of e^{rT} . *i.e.*, $e^{rT} \approx 1 + rT + \dfrac{(rT)^2}{2}$

Hence equation (1.12) reduces to $Ar - \left(CD + \dfrac{hD}{r}\right)\dfrac{(rT)^2}{2} = 0$

And consequently, we have the approximate replenishment cycle time

$T^* \approx \sqrt{\dfrac{2A}{(CDr + hD)}}$ And approximate optimal order quantity

$$Q^* = DT^* \approx \sqrt{\frac{2DA}{(Cr+h)}}$$

4.1.5 Computation of Optimality when the Late Payment Penalty is Imposed from the Purchased Date

In certain business transactions, if a customer is not able to make the payment within the due date, then a penalty will be charged from the day of purchasing that item instead of from due date. In the case of trade credit also we can examine the optimal solution if a penalty is charged from the date of purchase. It is clear that in this situation profit will be higher compared to the profit obtained in the above situation (11). We will consider that penalty rate r_p which is charged from the time of purchasing an item, hence, this rate may take a different value compared to the earlier r_p.

The net expected present value of the sales revenue for all the cycles in the planning horizon for delay in payments is derived similarly to (3) and is

$$\sum_{1}^{\infty}\int_{0}^{T}\int_{M}^{\infty} SDe^{-r(t+(n-1)T+u)} e^{r_p(u)} g(u-M) \quad du\, dt$$

$$= \frac{SD}{r}\int_{0}^{\infty} e^{-r(v+M)}\; e^{r_p(v+M)} g(v)dv$$

$$= \frac{SDe^{(r_p-r)M}}{r}\left[\int_{0}^{\infty} e^{v(r_p-r)}\; g(v)dv\right]$$

By referring (4), above expression reduces to $\dfrac{SDe^{(r_p-r)M}\theta^{\alpha}}{r(\theta+r-r_p)^{\alpha}}$ (13)

Using the expression (1), (2), (8), (10) and (13) the profit function is,

$$Z_2(\mathrm{T}) = \xi\left(\frac{SD}{r}p_1 + \frac{SD}{Mr^2}\left(1-e^{-rM}\right)p_2 + \frac{SDe^{(r_p-r)M}\theta^{\alpha}}{r(\theta+r-r_p)^{\alpha}}p_3\right)$$
$$+ (1-\xi)\frac{SD}{r} - \left(\frac{A}{(1-e^{-rT})} + \frac{CDT}{(1-e^{-rT})} + \frac{hDT}{r(1-e^{-rT})} - \frac{hD}{r^2}\right) \tag{14}$$

The optimal cycle time and optimal order size are same as derived earlier in the expression (12). Substituting the resultant value of T* in the equation (14), we obtain the total optimum profit value when penalty charges for late payment from the day of purchasing an item.

4.2 Model B: The Trade Credit Period Coincides with the Replenishment Cycle Period

In this model, it is considered that the retailer may fix the trade credit duration equal to the optimal replenishment period. Hence reliable customers are given trade credit exactly up to T (the replenishment point) irrespective of their buying time, if payment goes beyond that, they will be charged penalty at the rate of r_p. We assume that probability that payment made at the time of purchasing an item (u=0) is p_1, probability that payment made within trade credit period M (replenishment cycle period) is p_2 and probability that payment made after the replenishment point with a penalty rate r_p is p_3. Any distribution which has nonnegative variable can be considered for payment time beyond the replenishment cycle period. It depends on the payment pattern by the reliable customers. One of the appropriate distributions for the delay in payment beyond the replenishment cycle period is gamma distribution. In our discussion here, we assume this distribution and derive the profit function.

4.2.1 The Expected Net Present Value of Sales Revenue for the Reliable Customers

The net present value of sales revenue per unit item per unit demand at various time points of payment when demand occurs at time t in the n[th] period, ($0\leq t \leq T$), includes three components

1. Sales revenue of all future cash flow when payment made at the time of purchasing an item (u=0) with probability p_1 ,
2. Sales revenue of all future cash flow when payment made with delay time of u with $0 < u \leq$. (T-t), *i.e.*, within the replenishment period. This is with probability p_2. Payment during this period could occur at any random time point and assume payment time during this interval follows uniform random variable say 'u'. i.e., 'u' follows a uniform distribution (0, T-t].
3. Sales revenue of all future cash flow when payment made after the replenishment period with probability p_3. Here penalty rate r_p is charged for late payment from the time of replenishment period.

$$g(t,u) = \begin{cases} Se^{-r(t+(n\text{-}1)T)} & u = 0 \\ Se^{-r(t+(n\text{-}1)T+u)} & 0 < u \leq T - t \\ Se^{-r((n\text{-}1)T+t+u)} \quad e^{r_p(t+u-T)} & u \geq T\text{-}t \end{cases}$$

Hence expected net present value of sales revenue for n[th] period is

$$SR(T,n) = \int_0^T SD\left[p_1\, e^{-r(t+(n-1)T)} + p_2 \int_0^{T\text{-}t} e^{-r(t+(n-1)T+u)}\; f(u)du + \right.$$

$$\left. p_3 \int_{T\text{-}t}^{\infty} e^{-r((n-1)T+t+u)} e^{r_p(t+u-T)} g(t+u-T)du\right] dt$$

The expected net present value of sales revenue for n^th period when payment made at u=0 is

$$\int_0^T SDe^{-r\,(t+(n\text{-}1)T)}dt = SDe^{-r\,(n\text{-}1)\,T}\frac{\left(1-e^{-rT}\right)}{r}$$

The expected net present value of the sales revenue for all the cycles in the planning

Horizon when payment made at u=0 is $\sum_1^{\infty} SDe^{-r\,(n\text{-}1)\,T}\frac{\left(1-e^{-rT}\right)}{r}=\frac{SD}{r}$ (15)

The expected net present value of sales revenue per n^th period when payment is made within the replenishment period is

$$= \int_0^T\int_0^{T-t} sDe^{-r(\,t+(n\text{-}1)T+u\,)}f(u)\,du\,dt$$

$$= \int_0^T \frac{1}{T-t}SDe^{-r\,(t+(n\text{-}1)\,T\,)}\left(\int_0^{T-t}e^{-ru}du\right)dt$$

$$= \frac{SDe^{-rTn}}{r}\int_0^T\frac{(e^{ry}-1)}{y}dy$$

$$= \frac{SDe^{-rTn}}{r}\int_0^T\frac{1}{y}\left(ry+\frac{(ry)^2}{2!}+\frac{(ry)^3}{3!}+\ldots\right)dy$$

$$= \frac{SDe^{-rTn}}{r}\left(\frac{rT}{1}+\frac{(rT)^2}{2.2!}+\frac{(rT)^3}{3.3!}+\ldots\ldots.\right)$$

The expected net present value of the sales revenue for all the cycles in the planning horizon when payment is made within the replenishment period is

$$\sum_1^{\infty}\frac{SDe^{-rTn}}{r}\left(\frac{rT}{1}+\frac{(rT)^2}{2.2!}+\frac{(rT)^3}{3.3!}+\ldots..\right)$$

$$= \frac{SD}{r}\left(\sum_1^{\infty}\frac{(rT)^n}{n.n\,!}\right)\sum_1^{\infty}e^{-rnT}$$

$$=\frac{SD}{r}\left(\sum_{1}^{\infty}\frac{(rT)^n}{n.n\,!}\right)\left(\frac{e^{-rT}}{(1-e^{-rT})}\right) \tag{16}$$

The expected net present value of sales revenue per n^th period when payment made after the replenishment period with penalty rate r_p is (Penalty is charged from trade credit point)

$$\int_{0}^{T}\int_{T\text{-}t}^{\infty} SDe^{-r(t+(n\text{-}1)T+t+u)}e^{r_p(t+u-T)}g(t+u\text{-}T) \quad du\,dt \tag{17}$$

$$=\int_{0}^{T} SDe^{-r(n\text{-}1)T}e^{-rt}e^{-rT}\left(\int_{0}^{\infty} e^{v(r_p-r)}\;g(v)\,dv\right)dt$$

$$=\int_{0}^{T} SDe^{-rnT}e^{-rt}\;M_Y(r_p-r)\,dt \tag{18}$$

$$where \quad M_Y(t)=\int_{0}^{\infty} e^{tv}g(v)\,dv$$

By referring (5) the equation (18) reduces to

$$=\int_{0}^{T} SDe^{-rnT}e^{-rt}\;\frac{\theta^{\alpha}}{(\theta+r-r_p)^{\alpha}}dt$$

$$=\frac{SDe^{-rnT}\theta^{\alpha}}{r(\theta+r-r_p)^{\alpha}}(1-e^{-rT})$$

Expected net present value of the sales revenue for all the cycles in the planning horizon when payment made after the replenishment period with a penalty rate r_p is

$$=\sum_{1}^{\infty}\frac{SDe^{-rnT}\theta^{\alpha}(1-e^{-rT})}{r(\theta+r-r_p)^{\alpha}}$$

$$=\frac{SD\theta^{\alpha}(1-e^{-rT})}{r(\theta+r-r_p)^{\alpha}}\left(\frac{1}{(1-e^{-rT})}-1\right)$$

$$= \frac{SD\theta^{\alpha} e^{-rT}}{r(\theta + r - r_p)^{\alpha}} \tag{19}$$

Using (15), (16) and (19) net expected present value of the sales revenue for all the cycles in the planning horizon for different time periods with assigned probabilities is

$$= \frac{SD}{r} p_1 + \frac{SD}{r}\left(\sum_{1}^{\infty} \frac{(rT)^n}{n.n\,!}\right)\left(\frac{e^{-rT}}{(1-e^{-rT})}\right) p_2 + \frac{SDe^{-rT}\theta^{\alpha}}{r(\theta + r - r_p)^{\alpha}} p_3 \tag{20}$$

Using (19) and (20) expected sales revenue for different time points if general as well as reliable customers considered together with respective probabilities ξ and (1- ξ) are

$$\xi\left(\frac{SD}{r} p_1 + \frac{SD}{r}\left(\sum_{1}^{\infty} \frac{(rT)^n}{n.n\,!}\right)\left(\frac{e^{-rT}}{(1-e^{-rT})}\right) p_2 + \frac{SDe^{-rT}\theta^{\alpha}}{(\theta + r - r_p)^{\alpha}} p_3\right) + (1-\xi)\frac{SD}{r} \tag{21}$$

4.2.2 Profit Function and Optimal Replenishment Cycle Time

The total cost is already obtained and shown in an expression (10)

Using (10) and (19), we express profit function as the difference between sales revenue and total cost, and is Z_3 (T, n) =

$$\xi\left(\frac{SD}{r} p_1 + \frac{SD}{r}\left(\sum_{1}^{\infty} \frac{(rT)^n}{n.n\,!}\right)\left(\frac{e^{-rT}}{(1-e^{-rT})} p_2\right) + \frac{SDe^{-rT}\theta^{\alpha}}{r(\theta + r - r_p)^{\alpha}} p_3\right)$$
$$+ \frac{SD(1-\xi)}{r} - \left(\frac{A}{(1-e^{-rT})} + \frac{CDT}{(1-e^{-rT})} + \frac{hDT}{r(1-e^{-rT})} - \frac{hD}{r^2}\right) \tag{22}$$

To obtain the optimal value of T, we maximize the profit function Z_3 (T), at the optimal value of T the profit function has zero derivative and second order derivative is negative.

Which means the condition to maximize Z_3 (T) is respectively,

$\dfrac{dZ_3(T)}{dT} = 0, \qquad \dfrac{d^2 Z_3(T)}{dT^2} \le 0$, hence we obtain,

$$\Rightarrow \frac{d}{dT}\left\{\begin{array}{l} SD\xi\left[\dfrac{e^{-rT}}{\left(1-e^{-rT}\right)}\right]\left[\dfrac{1}{r}\left(\sum_{1}^{\infty}\dfrac{(rT)^n}{n.n\,!}\right)p_2+\dfrac{\theta^{\alpha}\left(1-e^{-rT}\right)}{r(\theta+r-r_p)^{\alpha}}p_3\right] \\ -\left[\left(\dfrac{A}{(1-e^{-rT})}\right)+\left(\dfrac{CDT}{(1-e^{-rT})}\right)+\left(\dfrac{hDT}{r(1-e^{-rT})}-\dfrac{hD}{r^2}\right)\right] \end{array}\right\}=0$$

After simplification,

$$SD\xi\left(\frac{e^{-rT}}{\left(1-e^{-rT}\right)^2}\right)\left(\sum_{1}^{\infty}\frac{(rT)^n}{n\,!}\frac{\left(1-e^{-rT}\right)}{rT}-\sum_{1}^{\infty}\frac{(rT)^n}{n.n\,!}\right)p_2-SD\xi\left(\frac{e^{-rT}}{\left(1-e^{-rT}\right)^2}\right)\left(\frac{\theta^{\alpha}}{(\theta+r-r_p)^{\alpha}}\right)(1-e^{-rT})^2p_3$$

$$-\left(\frac{-Are^{-rT}}{(1-e^{-rT})^2}\right)-\frac{CD\left(1-e^{-rT}-rTe^{-rT}\right)}{(1-e^{-rT})^2}-\frac{hD}{r}\frac{(1-e^{-rT}-rTe^{-rT})}{(1-e^{-rT})^2}=0$$

$$SD\xi\left(\left(\frac{(e^{rT}-1)(1-e^{-rT})}{rT}-\sum_{1}^{\infty}\frac{(rT)^n}{n.n\,!}\right)p_2-\frac{\theta^{\alpha}(1-e^{-rT})^2}{\left(\theta+r-r_p\right)^{\alpha}}p_3\right)+Ar-\left(CD+\frac{hD}{r}\right)(e^{rT}-rT-1)=0 \tag{23}$$

Also, we get, $\dfrac{d^2Z_3(T)}{dT^2}\leq 0$,

Hence the optimal order size and replenishment time are derived by solving the equation (23).

$$SD\xi\left(\left(\frac{(e^{rT}-1)(1-e^{-rT})}{rT}-\sum_{1}^{\infty}\frac{(rT)^n}{n.n\,!}\right)p_2-\frac{\theta^{\alpha}(1-e^{-rT})^2}{\left(\theta+r-r_p\right)^{\alpha}}p_3\right)+Ar-\left(CD+\frac{hD}{r}\right)(e^{rT}-rT-1)=0$$

Substituting the resultant value of T* in the equation (22), we obtain the total optimum profit value

4.2.3 Optimality when Late Payment Penalty is Imposed from the Purchased Date: It can also be examined that the optimal solution of the above model if penalty is charged from the date of purchase rather than from the replenishment point. By using (17) net expected present value of the sales revenue for all the cycles in the planning horizon for delay in payments is

$$\sum_{1}^{\infty}\int_{0}^{T}\int_{M}^{\infty}SDe^{-r(t+(n-1)T+t+u)}e^{r_p(t+u)}g(t+u-T)\quad du\,dt$$

$$= \sum_{1}^{\infty} \int_{0}^{T} SDe^{-r(n\text{-}1)T} e^{-rt} e^{T(r_p - r)} \left(\int_{0}^{\infty} e^{v(r_p - r)} \, g(v) \, dv \right) dt$$

$$= \frac{SD}{r} e^{T(r_p - r)} \; M_Y(r_p - r) \, dt$$

$$Where \quad M_Y(t) = \int_{0}^{\infty} e^{tv} g(v) \, dv$$

By referring (5), equation (18) reduces to

$$= \frac{SD}{r} e^{T(r_p - r)} \frac{\theta^{\alpha}}{(\theta + r - r_p)^{\alpha}}$$

$$= \frac{SDe^{T(r_p - r)} \theta^{\alpha}}{r(\theta + r - r_p)^{\alpha}} \tag{24}$$

Using the equations (8), (15), (16), (10), and (24) the profit function is Z_4 (T, n) =

$$\xi \left(\frac{SD}{r} p_1 + \frac{SD}{r} \left(\sum_{1}^{\infty} \frac{(rT)^n}{n.n\,!} \right) \left(\frac{e^{-rT}}{(1 - e^{-rT})} p_2 \right) + \frac{SDe^{T(r_p - r)} \theta^{\alpha}}{r(\theta + r - r_p)^{\alpha}} p_3 \right) + \frac{SD(1 - \xi)}{r} - \left(\frac{A}{(1 - e^{-rT})} + \frac{CDT}{(1 - e^{-rT})} + \frac{hDT}{r(1 - e^{-rT})} - \frac{hD}{r^2} \right) \tag{25}$$

To obtain optimal value of T, we maximize the above profit function at the optimal value of T the profit function has zero derivative and second order derivative is negative.

Hence $\dfrac{dZ_4(T)}{dT}$ =0 gives

$$SD\xi \left[\left(\frac{(e^{rT} - 1)\,(1 - e^{-rT})}{rT} - \sum_{1}^{\infty} \frac{(rT)^n}{n.n\,!} \right) p_2 - \frac{\theta^{\alpha} (r_p - r) e^{r_p T} (1 - e^{-rT})^2}{r\left(\theta + r - r_p\right)^{\alpha}} p_3 \right] - \left[\left(CD + \frac{hD}{r} \right) (e^{rT} - rT - 1) - Ar \right] = 0 \tag{26}$$

Also, we get $\dfrac{d^2 Z_4(T)}{dT^2} \leq 0$,

Substituting the resultant value of T in the equation (25), we obtain the total optimum profit value when a late payment penalty is charged from the purchased date.

The optimal values of the decision variables and total profit were computed using MATLAB Programs and are presented in the following tables for different parameters.

4.3 Numerical Examples and Interpretations

To illustrate and verify the above theoretical results, we consider different situations in which probabilities of payment pattern differ for different trade credit periods.

Example 1: Demand rate per year, D=1000 units; interest rate, r =0.1; purchase cost, C =50; carrying cost, h =2/unit/item; ordering cost, A = 500/order; penalty rate, r_p=0.25; sales revenue s= 80; the distribution of delay beyond trade credit is assumed to be gamma distribution with parameters, α =0.1; θ =2

Following table shows the optimum order quantity, optimum replenishment cycle period and profit value with respect to the above parameters when a penalty is charged after the replenishment period and penalty is charged from the time of items received.

1. Though quantity Q does not vary for different trade credit limits, payment pattern has a significant impact on Profits. It may be noted that even when the order quantity remains same, the profits will depend on payment patterns of customers as well as transaction made by percentage of reliable customers and general customers. It can also be seen that the optimum replenishment cycle period remains same for given parameters even though trade credit period as well as customers' payment pattern differs. (0.3145 Years or 115 days)

Table 1 *Optimal Solutions when Trade Credit Period is Independent of the Replenishment Cycle Period*

Nature of Penalty		**Penalty charged after trade credit**		**Penalty charged from the beginning**	
Trade credit period (in Year)		M=0.137 (50 days)	M=0.22 (80 days)	M=0.137 (50 days)	M=0.22 (80 days)
Optimum Order quantity		314.52	314.52	314.52	314.52
Optimum replenishment cycle period		115 days	115 days	115 days	115 days
Cases	Probabilities	Profit			
1.	p_1=0.1, p_2=0.7, p_3=0.2, ξ=0.5	164545.82	162293.10	166599.02	165583.47

2.	p_1=0.4, p_2=0.4, p_3=0.2, ξ=0.5	165568.64	163931.07	167621.84	167221.44
3.	p_1=0.2, p_2=0.4, p_3=0.4, ξ=0.5	164458.03	162003.07	168616.60	168583.81
4.	p_1=0.2, p_2=0.4, p_3=0.4, ξ=0.7	163024.04	159587.10	168846.00	168800.13
5.	p_1=0.4, p_2=0.2, p_3=0.4, ξ=0.7	163978.68	161115.86	169727.64	170328.90

1. As we know that it is profitable for the retailer to charge penalty from the time of items delivered. The same is observed in Case 1.
2. By comparing Case 4 and Case 5, we can observe that when the proportion of late payment beyond trade credit is remaining same, indicates that customer is more likely to pay against delivery compared to the trade credit point that incurs more profits.
3. By comparing Case 3 and Case 4, we can observe that profit is less when proportion of reliable customer is larger compared under the same payment pattern of reliable customer.
4. It can be observed from Case 1 and Case 2 that profit is more when the customer is more likely to pay against delivery.

Example 2: Demand rate per year, D=1000 units; Interest rate, r =0.1; Purchase Cost, C =50; Carrying cost, h =2/unit/item; Ordering cost, A = 500/order; Penalty rate, r_p=0.25; Sales revenue s=80; the distribution of delay beyond trade credit is assumed to be gamma distribution with parameters, α =0.1; θ =2

Table 2 *Optimal Solutions when Trade Credit Period Coincided with the Replenishment Cycle Period*

Nature of Penalty charged		**Penalty charged after replenishment period**		**Penalty charged from the beginning**	
Cases	Probabilities	Optimal Quantity	Optimal Profit	Optimal Quantity	Optimal Profit
1.	p_1=0.1, p_2=0.7, p_3=0.2, ξ=0.5	248.65	165110.06	309.13	166526.97
2.	p_1=0.4, p_2=0.4, p_3=0.2, ξ=0.5	245.81	167321.96	310.38	167463.16
3.	p_1=0.2, p_2=0.4, p_3=0.4, ξ =0.5	249.53	162128.25	313.08	168977.69
4.	p_1=0.2, p_2=0.4, p_3=0.4, ξ =0.7	252.41	156893.96	313.28	169690.01
5.	p_1=0.4, p_2=0.2, p_3=0.4, ξ =0.7	253.14	158530.61	321.04	170563.79

1. As discussed in the Table 1.1, here also, we can observe that total profit is more when penalty is charged from the beginning rather than penalty charged after the

replenishment period. We can also notice that the optimal order quantity is more when penalty is charged from the beginning rather that penalty charged after the replenishment period.

2. The optimum order quantity is higher when the proportions of reliable customers are more compared to general customers.
3. When a penalty is charged from the beginning, profit is more in case of less number of reliable customers pay against replenishment, and more number of reliable customers paying after the replenishment period. Whereas when a penalty is charged after a replenishment period, profit is less in case of less number of reliable customers pay against replenishment, and more number of reliable customers paying after the replenishment period.

5. Conclusion

In this paper, we have developed optimal policies for the inventory model based on the nature of the customers. Customers, who are known to retailers, regularly do business with the retailer and very prompt in making the payments are called as reliable customers and they are given trade credits. The other customers to whom retailer demands immediate payment and does not apply trade credit are called as general customers. In addition, we have characterized optimal solution when trade credit is independent of replenishment cycle period and also trade credit period coincides with the replenishment cycle period. The solution to the optimal credit period for the retailer when trade credit period coincides with the replenishment cycle period is also obtained. Optimum solutions are also determined separately when a penalty is charged for the trade credit period and from the time of items are delivered to the customers. Finally, we have provided some numerical examples to illustrate the proposed model and its optimal solution.

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