Optimal Decisions for an Impresario before the Start of Program Series

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This study is carried out to obtain optimal policies for an impresario who organizes various entertainment programs. The impresario brings an artist or group of artists which includes well-known local or national or international artist/s to perform before the audience. Impresario starts ticket booking in advance and uses frequent advertisements during the cycle time T. Impresario books an auditorium and artist/s for the number of days depending on the number of tickets sold and the capacity of the auditorium. Ticket demand depends on the selling price of tickets, quality of artists, and advertisement frequency. The proposed study aims to maximize the total profit of the impresario.

Keywords: Advance Booking; Advertisement; Artist; Impresario; Program Series; Selling Price

1. Introduction

The talent program industry has grown enormously in the past decade. The talent industry is an ocean of artists with different abilities and talents. These artists perform their flair on a stage to entertain people and earn money. An impresario is a bridge between such artists and the audience. In recent years, event management has become essential to bring various artists on a stage and to make the program successful. This system is comprised of three market players viz. artist/s, an auditorium owner, and the impresario. Customers of the system are the audience that comes to watch and enjoy the program. Amongst these three players artist/s charge a fixed amount of money for each performance and the auditorium owner also charge a fixed amount as rent for each program. These two players do not put any extra effort to bring customers to the system. Thus artist/s and the auditorium owner behave as passive players of the system. On the other hand, the impresario puts effort and manages to bring different well-known artist/s at local or national, or international levels to perform in a program series. The impresario uses frequent advertisements to promote the advance booking of the tickets for the programs to gather an audience for shows. Thus the impresario behaves as an active member of the system. Hence the study has been carried out by keeping the impresario in the center.

The ticket booking system plays an important role for inventory systems with a limited number of seats. Tak and Marvin (1993) studied airline seat inventory control with multiple seat bookings. They developed a dynamic programming model with

discrete-time for finding out an optimal booking policy. Advance booking is a must part of program series events. It gives an actual idea of the arrangement of a number of performances to be arranged by taking into account the total capacity of the auditorium and the total number of tickets sold in the advance booking phase. Aggarwal *et al* (2020) investigated optimal policies to maximize the total profit with considering of an advance booking system. They obtained optimal order quantity, selling price, and advertisement expenditure. Shah *et al*. (2021) gave optimal inventory policies for deteriorating products with an advanced booking system. The quality of the product is helpful to bring in creamy layer customers. Some customers emphasize more on the product's quality. Cynthia *et al*. (2021) developed an inventory model with price-sensitive demand for growing products with imperfect quality. They considered the effect of carbon emissions and allowed shortages in their study. Datta (2013) investigated an inventory system where some items produced are defective and gave optimal inventory decisions under price and quality-dependent product demand.

An advertisement is a tool that a product seller uses for the enhancement of product sales by creating awareness of the product in the market. For many products, frequent advertisements bring in more customers. The selling price of the product also affects the sales of the product. A higher selling price reduces product demand and on the other side lower selling price may increase product demand but it results in lower sales revenue. Cardenas Barron and Sana (2015) used promotional efforts that affected demand to obtain economic order quantity for a multi-item inventory model. Saha *et al*. (2019) gave optimal inventory policies for a three-level supply chain with a manufacturer, a distributor, and a retailer. They considered a service level agreement in which they offered a reasonable price, quality product, timely delivery, and price rebate to the customer. Dye (2020) analyzed the dynamic advertisement problem with the effect of the psychic stock level. In the study they considered joint pricing and advertising to obtain optimal policies for perishable products. Navarro *et al*. (2020) gave a collaborative economic production quantity for a three-echelon supply chain with multiple products considering the effect of a marketing effort on demand. Shaikh and Cardenas (2020) developed an inventory model for non-instantaneous deteriorating items with price-sensitive and advertisement-affected demand. Shah *et al*. (2021) gave optimal inventory policies for a retailer for a deteriorating items inventory model with preservation technology investments. They considered promotional efforts to increase product sales. Udaykumar *et al*. (2021) presented an inventory model for deteriorating items with a trade credit policy. They considered price-sensitive and advertisement-dependent product demand.

This study considers three market players. Artist/s and an auditorium owner are passive players while an impresario is an active player. Customers in the system are the audience for the shows. The impresario brings well-known artist/s at the local level or national level or international level as per public demand and craze. The impresario uses frequent advertisements as a promotional tool to make people aware of the program series and to promote sales of the tickets of shows in the advance booking phase $[0, T]$. Constant demand 'a' is affected by the popularity parameter α, selling price P of tickets, and advertisement frequency F. Once the advance booking phase ends, the impresario books artist/s and an auditorium on rent for 'N'

number of days, where N depends on the total number of tickets sold in advance booking phase and the total capacity of the auditorium. The impresario pays rent for N days to the auditorium owner. The impresario bears the traveling cost, lodging cost, and boarding cost of artist/s and pays remuneration for N days.

The study has been accomplished with the research questions mentioned below

1. Do the frequent advertisement policy useful to increase sales of the tickets?
2. What is the impact of the popularity parameter of artist/s on sales of tickets and the total profit of the impresario?

With these focused research questions, we aim to find out the optimal advance booking time, selling price of tickets, and advertisement frequency for the impresario to make the program series successful in terms of total sales of tickets and total profit.

The remaining part of the study is organized as section 2 includes assumptions and notations used for the development of the mathematical model. The mathematical model and solution procedure are explained in section 3. Section 4 is comprised of numerical examples and sensitivity analyses. Section 5 concludes the paper.

2. Assumptions and Notation

The authors will use the following abbreviations in the development of the proposed model.

2.1 Abbreviations

Parameters associated with artist/s

α Quality parameter associated to popularity or brand value of artist/s $(0 \le \alpha \le 1)$

A_0 Base value of traveling expenses of artist/s (in \$)

A_1 Actual value of traveling expenses of artist/s (in \$) $\left[A_1 = (1+\alpha) A_0\right]$

h_0 Base value of lodging-boarding expenses of artist/s (in \$)

h_1 Actual value of lodging and boarding expenses of artist/s per day (in\$) $\left[h_1 = (1+\alpha) h_0\right]$

C_0 Base price charged by artist/s per show (in \$)

C_1 Actual price charged by artist/s per show (in \$) $\left[C_1 = (1+\alpha) C_0\right]$

Parameters associated with auditorium owner:

S Total number of seats available in the auditorium or capacity of the auditorium

C_2 Cost of electricity, audio system, decoration per day (in \$)

A_2 Set up cost for auditorium owner (in \$)

h_2 Maintenance and cleaning cost per seat per day (in $)

X Rent charged by the owner to impresario per day (in $)

TC_2 Total cost (in $) of the auditorium owner for N days

Parameters associated with impresario

$I(t)$ Number of unsold tickets at time t

T Cycle time of impresario for advance booking of show tickets

A Set up cost for impresario (in $)

h Cost like office expenditure and other expenses per day till booking cycle time T

HC Total holding cost of impresario (in $)

N Number of days for which auditorium and artist/s are booked

M Advertisement cost per advertisement (in $)

F Advertisement frequency during the advance booking phase

ψ Advertisement effectiveness factor $(0 \le \psi \le 1)$

Q Total number of tickets sold during the advance booking phase

P Selling price of a single ticket for a show (in $)

P_1 Total remuneration paid to the artist/s for N days (in $)

TC_1 Total amount paid to artist/s by the impresario (in $)

P_2 Total amount paid to auditorium owner by the impresario (in $)

TC Total cost of the impresario (in $)

R Ticket demand function

TP Total profit of impresario per unit cycle (in $) (Objective function)

2.2 Assumptions

1. In the advance booking phase interval $[0, T]$, the impresario sells tickets with the help of frequent advertisements.
2. Once the advance booking phase ends, the impresario books an auditorium and artist/s for N days. Here, $N = \frac{Q}{S}$ and it should be an integer. To retain continuity of the profit function, initially, we consider N as a continuous real variable to obtain the optimal value of decision variables and maximize the total profit using the second derivative test. Then we consider the ceiling value of N to obtain values of decision variables and the total of the impresario.
1. There is only one show per day and any show is not canceled for any reason.
2. Depending on public craze and demand the impresario brings local artist/s or national artist/s or international artist/s.

3. C_0 is the base price to be charged by artists per show. α is the quality parameter that decides the actual payment amount $\left[C_1 = (1+\alpha)C_0\right]$ to artist/s per show.
4. We consider $\alpha = 0$ for a local artist, $\alpha = 0.5$ for a national artist, and $\alpha = 0.75$ for an international artist.

3. Development of Mathematical Model

In this section, we now proceed to develop a mathematical model for the above-described problem to obtain optimal solutions and proper strategies for the impresario. Ticket demand is influenced by advertisement frequency, popularity parameter of artist/s, and selling price of the ticket. The demand function is as shown below in equation (1).

$$R = \left[a(1+\alpha) - b \cdot p\right] \cdot F^{\psi} \tag{1}$$

The impresario uses frequent advertisements to make people aware of the program series and to promote the sales of tickets during the advance booking period $[0, T]$. The total number of unsold tickets at any time *t* is governed by the differential equation as shown below

$$\frac{d}{dt} I(t) = -R,\ 0 \le t \le T. \tag{2}$$

We solve the differential equation (2) using the boundary condition $I(T) = 0$ and obtain the solution as represented in equation (3).

$$I(t) = R(T - t). \tag{3}$$

Moreover, the total tickets quantity or the total number of tickets to be sold during the advance booking period can be obtained using the initial condition

$I(0) = Q$. Hence using this condition in equation (3) we get, $Q = RT$. (4)

Depending on the total number of tickets sold during the advance booking period the impresario books an auditorium with maximum seating capacity *S* for *N* days.

Hence, $N = \frac{Q}{S}$. (5)

The impresario books artist/s for *N* days to perform one show per day in the program series. Further, apart from the remuneration per day, the impresario bears

traveling expenses, lodging, and boarding expenses of artist/s. Now we consider the total earnings of each player of the system individually.

3.1 Artist/s Perspective

Travelling cost of artist/s is $A_1 = (1+\alpha) A_0$ (6)

Lodging and boarding cost of artist/s for *N* days is $HC_1 = (1+\alpha) h_1 \cdot N$ (7)

Total remuneration of artist/s for *N* days is $P_1 = (1+\alpha) C_0 \cdot N$ (8)

However, in this system artist/s is considered as a passive player, as this player is concerned with the remuneration per day only. The impresario bears all the expenses of artist/s. Hence the total amount paid by the impresario to the artist/s is as shown below in equation (9).

$$TC_1 = A_1 + HC_1 + P_1 = (1+\alpha) A_0 + (1+\alpha) h_1 \cdot N + (1+\alpha) C_0 \cdot N \quad (9)$$

Total earnings of the artist/s is given by, $TP_A = P_1 \cdot N$ (10)

3.2 Auditorium Owner's Perspective

Set-up cost of the auditorium is A_2.

The daily cost of electricity, audio system, light system, decoration cost are considered under the head basic cost of the

auditorium owner which is given by, $CP_2 = N \cdot C_2$. (11)

The daily maintenance cost of the auditorium is $HC_2 = h_2 \cdot S \cdot N$ (12)

Hence, the total cost of the auditorium owner is as shown below in equation (13).

$$TC_2 = A_2 + HC_2 + CP_2 = A_2 + h_2 \cdot S \cdot N + C_2 \cdot N \quad (13)$$

The total revenue received by the auditorium owner from the rent of the auditorium for *N* days is

$$P_2 = X \cdot N \quad (14)$$

Hence, the total profit of the auditorium owner per day is

$$TP_o = \frac{1}{N}\left[P_2 - TC_2\right] = \frac{1}{N}\left[P_2 - \left(A_2 + h_2 \cdot S \cdot N + C_2 \cdot N\right)\right] \quad (15)$$

3.3 Impresario's Perspective

Set-up cost of booking office is A .

Total advertisement cost is $MC = F \cdot M$ (16)

Total holding cost which include rent, equipment and staff for booking office is given by,

$$HC = h \cdot \int_0^T R \cdot t \, dt = \frac{h \cdot R \cdot T^2}{2} \tag{17}$$

Total rent for the auditorium for N days is as shown in equation (12) and total payment to artist/s is as shown in equation (9).

Thus, the total cost of the impresario is as shown below.

$$TC = A + HC + MC + TC_1 + P_2 \tag{18}$$

Total revenue received through the sale of tickets is

$$TR = P \cdot Q \tag{19}$$

Hence the total profit of the impresario per unit cycle is as shown below in equation (20).

$$TP = \frac{1}{T+N}(TR - TC) \tag{20}$$

3.4 Solution Procedure

We follow the below-mentioned procedure to obtain the optimal solution.

Step-1: Allocate values of the parameters other than decision variables in equation (20).

Step-2: Differentiate equation (20) partially for decision variables *F, T,* and *P* respectively to obtain first-order partial

derivatives $\frac{\partial TP}{\partial F}, \frac{\partial TP}{\partial T}$ and $\frac{\partial TP}{\partial P}$ respectively.

Step-3: Set these first-order partial derivatives equal to zero and solve them to obtain the values of *F, T,* and *P*.

Step-4: Substitute the values of *F, T*, and *P* obtained in step-3 together with values of other inventory parameters in equation (4) and equation (20) to obtain the value of *Q* and total profit of impresario *TP* respectively. Use the value of *Q* in equation (5) to get the value of *N*.

In the next section, we provide a numerical example with hypothetic data to validate and explain the mathematical model.

4. Numerical Example and Sensitivity Analysis

4.1 Numerical Example

Example: Let $A = \$100,\ A_1 = \$500,\ A_2 = \$400,\ a = 1000,\ b = 0.5,$ $C_0 = \$10000,\ C_2 = \$2000,\ h = \$1.5$ per day, $h_0 = \$1000$ /day, $h_2 = \$0.5$ /seat per day, $S = 1000,\ X = \$15000,\ \psi = 0.01,\ M = \$2500$.

We consider $\alpha = 0, \alpha = 0.5$ and $\alpha = 0.75$ as popularity factor separately for local, national and international artist/s respectively.

Executing the procedure mentioned in section 3.4, we get optimal values of decision variables, total profit for different cases as shown below in Table 1.

Table 1 *Optimal Values of Decision Variables and Total Profit in Different Cases*

α	F	T	P	TP	Q	N
0	15.02	11.030 days	\$ 1185.72	\$ 337170.88	4614	4.61 days
0.5	41.66	15.363 days	\$ 1857.07	\$ 671124.89	9113	9.11 days
0.75	60.63	17.369 days	\$ 2205.64	\$ 864002.29	11712	11.71 days

Values of *F*, *T*, and *P* are independent of the value of *N*. Value of *N* cannot be a fraction. The impresario needs to pay the remuneration to artist/s and rent of auditorium to the owner for full-day thus the value of *N* affects the total cost of impresario and hence the total profit. The actual profit of the impresario considering the ceiling value of *N* (smallest integer less than or equal to *N*) is as shown below in Table 2.

Table 2 *Optimal Values of Decision Variables and Total Profit in Different Cases for Integer Values of N*

α	F	T	P	TP	Q	N
0	15.02	11.030 days	\$ 1185.72	\$ 328426.11	4614	5 days
0.5	41.66	15.363 days	\$ 1857.07	\$ 646521.58	9113	10 days
0.75	60.63	17.369 days	\$ 2205.64	\$ 855211.97	11712	12 days

Concavity of profit function for $\alpha = 0$ is shown in Figure-1. Similar concavity of the profit function can be seen for other values of α.

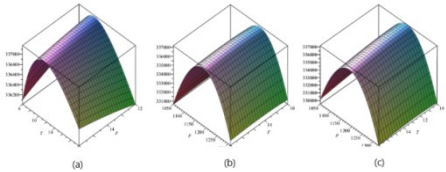

Figure 1 *Concavity of Total Profit for Decision Variables for* $\alpha = 0$

Figure 1 (a) represents the concavity of the profit function for T and F. Figure 1 (b) shows the concavity for P and F, while concavity for P and T is given in Figure 1 (c).

4.2 Sensitivity Analysis

Next, we proceed to check the sensitiveness of the decision variables and total profit for minor changes in other inventory parameters. To analyze the sensitivity we change each inventory parameter separately by $-20\%, -10\%, 10\%$ and 20%. The analysis for $\alpha = 0$ is represented below in Table 3.

Table 3 *Sensitivity Analysis of Decision Variables, Total Profit, Advertisement Cost, the Total Number of Tickets Sold, and Number of Days for a Change in other Inventory Parameters* $(\alpha = 0)$

Para-meter	% Change	F	T	P	TP	TP_o	Q	M	N
'A'	-20%	15.02	11.025	1185.72	337172.17	12413.27	4612	37553.71	4.61
	-10%	15.03	11.028	1185.72	337171.53	12413.29	4613	37563.40	4.61
	10%	15.03	11.033	1185.72	337170.25	12413.34	4615	37582.77	4.62
	20%	15.04	11.036	1185.72	337169.61	12413.36	4617	37592.44	4.62
'A1'	-20%	14.99	11.002	1185.71	337177.29	12413.09	4602	37475.99	4.60
	-10%	15.01	11.016	1185.72	337174.09	12413.20	4608	37524.60	4.61
	10%	15.05	11.045	1185.73	337167.70	12413.42	4620	37621.45	4.62
	20%	15.07	11.059	1185.73	337164.51	12413.54	4626	37669.69	4.63
'A2'	-20%	15.03	11.030	1185.72	337170.89	12430.65	4614	37573.09	4.61
	-10%	15.03	11.030	1185.72	337170.89	12421.98	4614	37573.09	4.61
	10%	15.03	11.030	1185.72	337170.89	12404.64	4614	37573.09	4.61
	20%	15.03	11.030	1185.72	337170.89	12395.98	4614	37573.09	4.61
'a'	-20%	8.43	9.189	929.93	226973.50	12372.81	3145	21072.46	3.14
	-10%	11.45	10.117	1056.83	280075.97	12396.16	3852	28626.42	3.85
	10%	19.18	11.928	1316.45	397959.41	12426.30	5427	47954.26	5.43
	20%	23.92	12.809	1448.89	462177.41	12436.38	6288	59802.82	6.29
'b'	-20%	23.57	13.731	1480.52	424745.88	12430.79	5779	58919.01	5.78
	-10%	18.58	12.229	1316.73	376058.49	12422.04	5131	46459.54	5.13
	10%	12.41	10.052	1078.56	305399.62	12404.61	4194	31016.12	4.19
	20%	10.42	9.240	989.28	278958.28	12395.95	3844	26042.02	3.84
'C1'	-20%	15.07	11.039	1185.02	337760.96	12413.46	4622	37669.73	4.62
	-10%	15.05	11.035	1185.37	337465.88	12413.39	4618	37621.39	4.62
	10%	15.01	11.026	1186.07	336875.99	12413.24	4610	37524.81	4.61
	20%	14.99	11.021	1186.42	336581.19	12413.16	4606	37476.58	4.61
'C2'	-20%	15.03	11.030	1185.72	337170.89	12813.31	4614	37573.09	4.61
	-10%	15.03	11.030	1185.72	337170.89	12613.31	4614	37573.09	4.61
	10%	15.03	11.030	1185.72	337170.89	12213.31	4614	37573.09	4.61
	20%	15.03	11.030	1185.72	337170.89	12013.31	4614	37573.09	4.61
'h'	-20%	18.75	13.741	1185.98	337714.76	12430.54	5759	46878.99	5.76
	-10%	16.68	12.235	1185.84	337427.83	12421.92	5123	41707.70	5.12
	10%	13.68	10.045	1185.61	336938.20	12404.73	4198	34191.97	4.20
	20%	12.55	9.223	1185.51	336725.54	12396.16	3852	31375.67	3.85
'h1'	-20%	15.03	11.031	1185.65	337229.88	12413.33	4615	37582.74	4.62
	-10%	15.03	11.031	1185.69	337200.38	12413.32	4615	37577.91	4.61
	10%	15.03	11.030	1185.76	337141.40	12413.31	4614	37568.26	4.61
	20%	15.03	11.030	1185.79	337111.90	12413.30	4613	37563.43	4.61

'h2'	-20%	15.03	11.030	1185.72	337170.89	12513.31	4614	37573.09	4.61
	-10%	15.03	11.030	1185.72	337170.89	12463.31	4614	37573.09	4.61
	10%	15.03	11.030	1185.72	337170.89	12363.31	4614	37573.09	4.61
	20%	15.03	11.030	1185.72	337170.89	12313.31	4614	37573.09	4.61
'X'	-20%	15.09	11.044	1184.67	338056.13	9413.53	4626	37718.11	4.63
	-10%	15.06	11.037	1185.20	337613.41	10913.42	4620	37645.56	4.62
	10%	15.00	11.024	1186.25	336728.58	13913.20	4608	37500.69	4.61
	20%	14.97	11.017	1186.77	336286.48	15413.09	4602	37428.38	4.60
'M'	-20%	18.81	11.027	1185.99	337707.08	12413.45	4622	37619.77	4.62
	-10%	15.03	11.030	1185.72	337170.89	12413.31	4614	37573.09	4.61
	10%	13.66	11.032	1185.61	336942.04	12413.25	4611	37553.14	4.61
	20%	12.51	11.034	1185.50	336733.20	12413.20	4608	37534.95	4.61

5. Conclusion

The present study is carried out to maximize the total profit of the impresario. The system is consisting of three players. Two members, the artist/s and the auditorium owner participate as passive members. On the other side the third member, the impresario plays a very active role in the system. The impresario uses frequent advertisements during the advance sales phase to create awareness of the program series and to increase total sales of tickets. Ticket demand is influenced by advertisement frequency, popularity parameter of artist/s, and selling price of tickets. The model is explained and validated through a numerical example and at the end, a sensitivity analysis has been worked out to examine the effect of minor changes in inventory parameters on decision variables and the total profit of the impresario.

The interesting results from the study fulfill the research aim by answering to research questions. The results show that frequent advertisements play a very important role to increase ticket sales. Further, from Figure 1 it can be observed that advertisement frequency up to a certain level is beneficial, and over advertising policy may cause a decrease in total profit. Moreover, the popularity parameter of artist/s is also a key parameter in tickets sale. The higher popularity of artist/s brings in more people to purchase show tickets and enhances the overall sales of tickets despite higher rates of tickets and eventually it results in an increase in the total profit of the impresario.

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